SECTION 5.3 – Annuities and Sinking Funds

So far, we have only discussed lump sum deposits and payments. However, it is not always possible to make a lump sum deposit or payment. Sometimes, making periodic payments or deposits is necessary.

Two common uses of Annuities:
1. saving money for large purchases such as a home, car, trip, college, etc.
2. retirement

Terminology:

Annuity – a sequence of equal payments made at equal periods of time

Two types of Annuities:
1. *ordinary annuity* – payments are made at the end of the time period
2. *annuity due* – payments are made at the beginning of the time period

Payment Period – time between payments

Term – time from first payment period to the end of the last payment period

Future Value of an Ordinary Annuity – final sum on deposit (sum of compound amounts of all payments) when payments are made at the end of the time period

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

where S is the future value,

R is the payment at end of each period,

$i = \frac{r}{m}$ (interest rate / period), and $n = mt$ (number of periods).

Example 1:

R = \$20,000, 6% interest compounded quarterly for 12 years. Find the future value of this ordinary annuity.

Example 2:

S = $43,000, interest is 6% compounded semiannually for 5 years. Find the periodic payment that will give this future value if payments are made at the end of each period.

Sinking Fund – a fund set up to receive periodic payments

If the payments are all the same and are made at the end of a regular time period, the sinking fund is essentially the same as an ordinary annuity.

Example 3:

Future Value is $6000; money earns 8% compounded monthly for 3 years. Find the amount of each payment to be made into a sinking fund.

Future Value of an Annuity Due – final sum on deposit (sum of compound amounts of all payments) when payments are made at the beginning of the time period

$$S = R\left[\frac{(1+i)^{n+1}-1}{i}\right] - R$$ where S is the future value,

R is the payment at beginning of each period,

$i = \frac{r}{m}$ (interest rate / period), and $n = mt$ (number of periods).

Example 4:

Payments of $1050 for 6 years at 3.5% compounded annually. Find the Future Value of this annuity due.