Material Balance – Sample Problem 1

A 10 gallon fish tank contains 2 percent salt. How much dry salt must be added to bring the salt concentration to 3.5 percent by weight?

Tips for Solving Problems

1. Write the Given and Find statements
2. Draw a picture of the system labeling all Inputs and Outputs for each component.
3. Assign variables to the unknowns. Use symbols to describe the known parameters and list their values.
4. Determine whether the problem is a batch process or rate process. Determine whether components are generated or consumed.
5. Write the conservation of mass for each component and for the entire system.
6. Use algebra to solve for the unknowns.
7. Check your work.

Note:

When working on a new problem, it is very unlikely that you will be able to follow the steps perfectly from the beginning. You will need to circle back or iterate on the steps. It is a good idea to work the problem on scrap paper first and then when the solution is obtained or when the solution process is obvious, transfer your preliminary work to the final version of your solution. Engineering problem-solving takes lots of practice.

Solution

1. Given and Find statements

   GIVEN:
   
   10 gallon fish tank
   Initial concentration: 2 percent salt
   Final concentration: 3.5 percent salt

   FIND:
   
   Amount of salt to be added

   Instructor Comment
   
   Do not simply restate or repeat the problem. Strip it down to its essential pieces
2. Draw a Picture

*Instructor Comment*

Pictures are very important, and it is hard to create a good schematic on the first try. Drawing different versions of the schematic is part of the problem-solving process. Here we show the picture in two stages. Only show the final schematic in your solution.

*Schematic to identify the process*

Start by identifying the high level process. There are two inputs: the initial salt water and the dry salt. There is one output: the final salt water mixture. We will add more detail to this schematic as we move through the problem-solving process.

3. Assign Variables to the Unknowns

**Parameters and Variables and Unknowns**

A **parameter** is a numerical quantity that is specified or known. Sometimes parameters are given as a range. In the example problem, the volume of water in the tank 10 gallons, is a parameter. Changing the parameter will change the result of the analysis, but the value of a parameter is known.

An **unknown** is a numerical quantity that you wish to find. You *solve for* an unknown. In the example problem, the amount of salt to be added is an unknown.

A **variable** is a symbol used to designate a numerical quantity. We assign variables to unknowns so that we can use algebra to solve for the unknown(s). Sometimes we assign a variable to a known parameter. In fact, that is often a good idea because it helps us to check the algebra in a solution.
There are two unknowns

\[ X = \text{mass of dry salt to be added} \]
\[ Y = \text{the total mass of the final mixture} \]

At the beginning of a typical engineering problem, the unknowns may not be obvious. Sometimes what appears to be an unknown is just a parameter that determines the outcome, but will be a constant for the given problem.

For example, in the current problem, why isn’t the mass of the water an unknown? It turns out the mass of water can be computed from the concentration of salt in the initial mixture. We could list the mass of water as an unknown, and then solve for it. That would be OK. However, to avoid making this sample problem too cluttered, we will simply note that the mass of water is a known parameter and let it show up in the analysis/solution of the problem.

There are several known parameters

\[ V_{\text{in}} = \text{volume of water in the initial mixture} \]
\[ C_{\text{in}} = \text{concentration of salt in the initial mixture (a known input)} \]
\[ C_{\text{out}} = \text{concentration of salt in the final mixture (the desired output)} \]

\( C \) is a fractional concentration. The mass percent (or weight percent) is \( 100 \times C \).

Concentration: Review of Units

Concentration is a dimensionless ratio. For example, the concentration of salt in saltwater is

\[
C_{\text{NaCl}} = \frac{m_{\text{NaCl}}}{m_{\text{NaCl}} + m_{\text{H}_2\text{O}}}
\]

Note that this assumes that salt water is a mixture of pure salt (NaCl) and pure water (H₂O). In fact, salt water has minute concentrations of many other minerals. We use a simple model that focuses only on the water and the salt.

Concentration is dimensionless. It is a ratio of masses. A proper ratio has the same dimensions in the numerator and denominator, so the result is dimensionless. The dimensions in the numerator cancel the dimensions in the denominator.

Problems involving concentration often have concentrations specified as weight percent. A percentage is just a fraction multiplied by 100. Therefore, we don’t need to create a new symbol for concentration when it is expressed as a percent. For example, a salt concentration of 3.5 percent can be written

\[ C = 0.035 \text{ (pure fraction, no dimensions, no percent sign)} \]

or

\[ C = 3.5 \% \]

How do we quantify the water in the initial and final mixtures?
The mass of the mixture is

\[ m_{\text{mixture}} = m_{H_2O} + m_{NaCl} \]

The concentration of water and salt are

\[ C_{H_2O} = \frac{m_{H_2O}}{m_{NaCl} + m_{H_2O}} \quad \quad \quad C_{NaCl} = \frac{m_{NaCl}}{m_{NaCl} + m_{H_2O}} \]

Rearrange as

\[ m_{H_2O} = C_{H_2O}(m_{NaCl} + m_{H_2O}) \quad \quad \quad m_{NaCl} = C_{NaCl}(m_{NaCl} + m_{H_2O}) \]

So

\[ m_{H_2O} + m_{NaCl} = C_{H_2O}(m_{NaCl} + m_{H_2O}) + C_{NaCl}(m_{NaCl} + m_{H_2O}) \]

Divide through by \( m_{H_2O} + m_{NaCl} \) to get

\[ 1 = C_{H_2O} + C_{NaCl} \]

Therefore

\[ C_{H_2O} = 1 - C_{NaCl} \]

This definition allows us to treat \( C_{NaCl} \) as the only unknown concentration.

2. Update the Picture

The preceding analysis involved very basic definitions that allow us to clearly identify the inputs and outputs. We know that the concentration of salt changes from initial to final state. We will use \( C_{in} \) and \( C_{out} \) to designate the initial and final salt concentrations. The unknowns \( X \) and \( Y \), and the known parameters \( V_{in}, C_{in} \) and \( C_{out} \) are added to the diagram.

Alternatively, we could use the numerical values for the parameters, as in the version of the system diagram on the following page. Note that \( X \) and \( Y \) are still variables.
4. Is this a Batch or a Rate Process
This is a batch process. We have a starting state with a known volume and concentration. We add salt to obtain an ending state. The material is not flowing continuously through the system. There are no species created or destroyed, i.e. there are no chemical reactions.

5. Write the Mass Balance for the system and each component.

**Overall Mass Balance**
The overall mass of salt and saltwater mixture is unchanged

\[ m_{in} = m_{out} \]  

(1)

The initial mass consists of the two ingredients: saltwater with a known concentration, and an unknown mass of salt. Thus, the left-hand side of Equation (1) is

\[ m_{in} = (m_{H_2O} + m_{NaCl})_{initial} + X \]  

(2)

To calculate the mass of the initial saltwater mixture, assume that the salt water has the same density as fresh water

\[ (m_{H_2O} + m_{NaCl})_{initial} = \rho_{H_2O}V_{in} = 62.3 \frac{lbm}{ft^3} \times 10 \text{gal} \times \frac{0.1337 ft^3}{gal} = 83.3 lbm \]  

(3)

So Equation (2) becomes

\[ m_{in} = 83.3 lbm + X \]  

(4)

From the schematic and the definition of the unknowns, \( m_{out} = Y \), therefore the overall mass balance is

\[ 83.3 lbm + X = Y \]  

(5)
Mass Balance for Salt

The total mass of salt before and after the mixing is unchanged

\[ m_{in} = m_{out} \]

For the salt alone this is

\[ C_{in}m_{in} + X = C_{out}Y \]

Or, substituting the known concentrations

\[ (0.02)(83.3lb_m) + X = 0.035Y \]

Mass Balance for Water

The total mass of water before and after the mixing is unchanged

\[ m_{in} = m_{out} \]

For the water alone this is

\[ (1 - C_{in})m_{in} + X = (1 - C_{out})Y \]

Or, substituting the known concentrations

\[ (0.98)(83.3lb_m) = 0.965Y \]

6. Use Algebra to solve for the unknowns

There are three equations and two unknowns. Normally the number of equations needs to equal the number of unknowns. However, Equation (6) and Equation (7) are redundant because \( C_{H_2O} = 1 - C_{NaCl} \).

Solve Equation (7) for \( Y \)

\[ Y = \frac{(0.98)(83.3lb_m)}{0.965} = 84.6lb_m \]

Substitute this result into Equation (6) and solve for \( X \)

\[ X = (0.035)(84.6lb_m) - (0.02)(83.3lb_m) = 1.24lb_m \]

Final solution

Mass of salt to be added: \( X = 1.24 lb_m \)

Mass of final mixture: \( Y = 84.6 lb_m \)

7. Check your work

Usually there is a way to verify that your computed result makes sense. In this case, we can test whether the overall mass balance holds by substituting the values of \( X \) and \( Y \) into Equation (5)

\[ 83.3lb_m + 1.29lb_m = 84.59lb_m \approx 84.6lb_m \]

This kind of check can help us avoid silly and potentially costly mistakes.