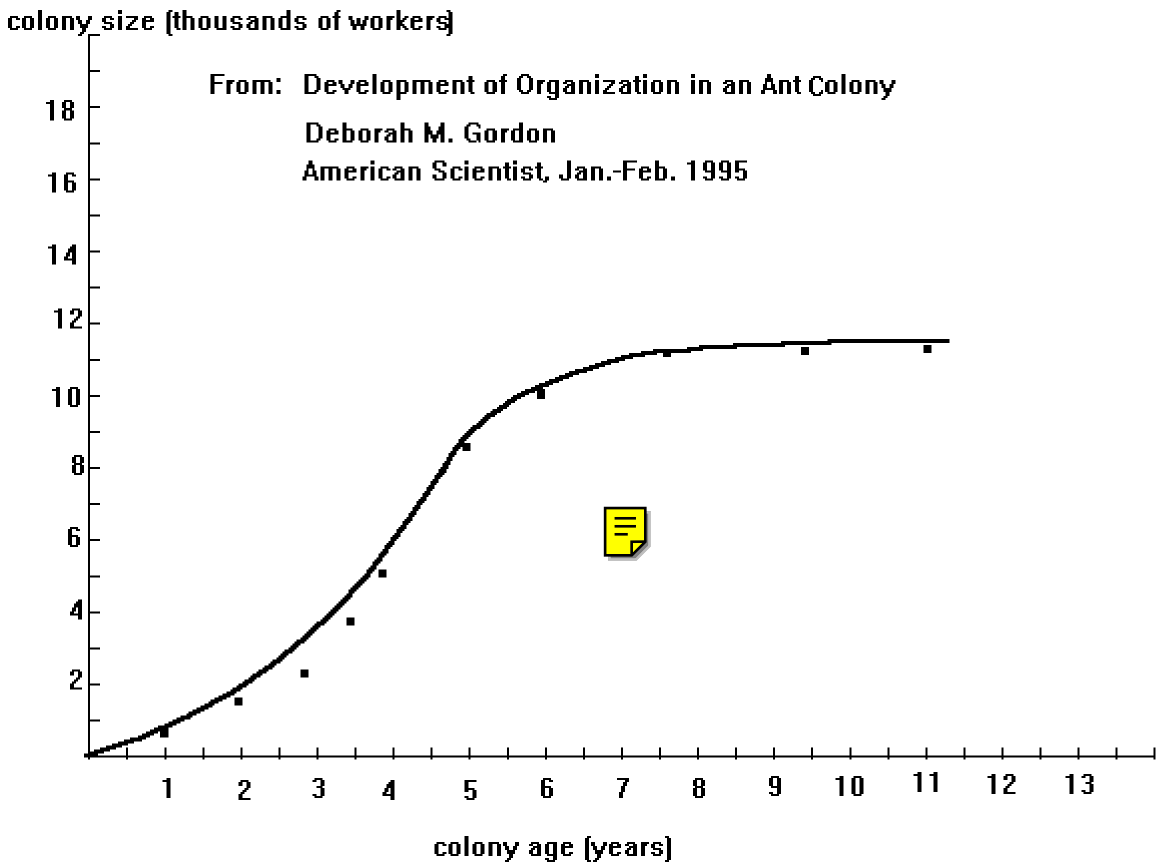
Ant Colony Model - The application of various population models to ant colony data

Problem based on: <u>The Development of Organization in an Ant Colony</u>
by: Deborah M. Gordon, American Scientist, Jan-Feb 1995

In the referenced article, a graph, reproduced below, is given, showing how an ant colony grows over time. Assuming the data given is representative of any number of ant colonies, you desire to develop a predictive equation which can be used to predict population growth in such colonies. You do a little research and find that the logistic population growth model is described by an equation of the form:

$$P(t) = \frac{P_L}{1 - c \cdot e^{-k \cdot t}}$$

where P , c and k are constants and P(t) is the population at time t. P_L represents the limiting value of the population since the limit of the population as time approaches ¥ = P_L, provided that k > 0. Use the data from the graph below in the table below to determine the constants P,c and k for a Logistic growth model for an ant colony. Graphically compare your model to the curve shown. Use your model to predict the limiting popuation of the colony. Compare your prediction to the actual value.



Introduction:

This problem will be done 2 ways, the first using a solve block and manually fitting the parameters; the second method uses the "genfit" algortihm.

Note: The numerical precedures for this problem are quite sensitive to the values of the initial estimates.

Method 1 - solve block

This method is based on the fact that, using the data given, we can write three equations in 3 unknowns. They are then solved iteratively using a solve block.

initial guesses for parameters to be estimated:

$P_L := 11100 \qquad c := -5497 \qquad k := 2.164$

When t = 0 years, we can write (minimum ants required...2): solve to get eqn. 1, below

$$2 = \frac{P_L}{1 - c \cdot \exp(-k \cdot 0)}$$

BEGIN SOLVE BLOCK

Given $\qquad P_L = 2 - 2 \cdot c \qquad$ equation 1

Take data from plot and use the same procedure to a similiar equation at 3 years

$$2200 \cdot \left(1 - c \cdot e^{-k \cdot 3}\right) = 2 - 2 \cdot c \qquad \text{equation 2}$$

Same procedure again...5 years

$$11000 = \frac{P_L}{(1 - c \cdot \exp(-k \cdot 5))} \qquad \text{equation 3}$$

unknowns are P_L, c and k

$$\begin{pmatrix} P_{Lsoln} \\ c_{soln} \\ k_{soln} \end{pmatrix} := \text{find}(P_L, c, k)$$

$$ERR = 0$$

$$\begin{pmatrix} P_{Lsoln} \\ c_{soln} \\ k_{soln} \end{pmatrix} = \begin{pmatrix} 1.137 \times 10^4 \\ -5.685 \times 10^3 \\ 2.406 \end{pmatrix}$$

solution vector

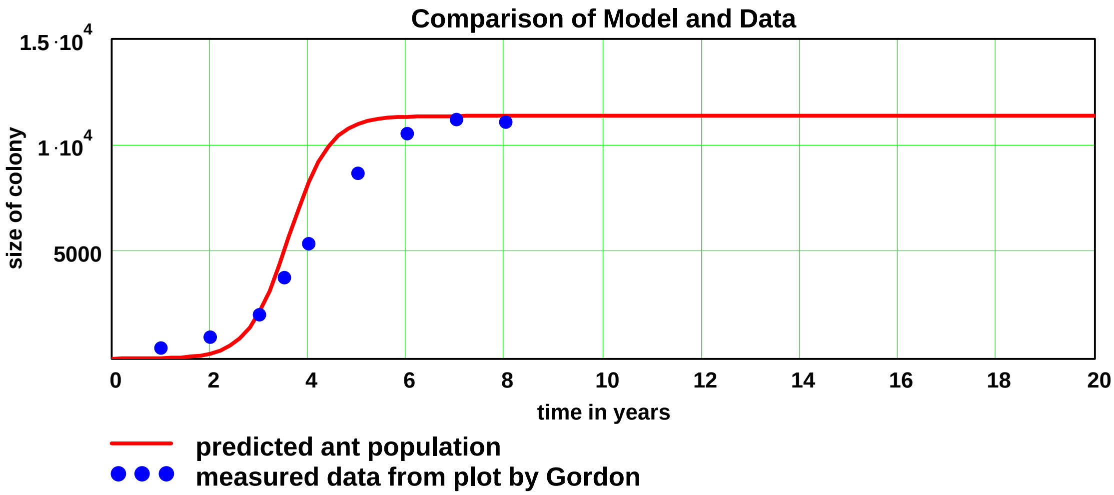
Now use the computed values of the parameters in the equation and compare equation predictions to the actual data. Since the computed values of the parameters depend on the initial guesses the initial guesses can be varied to try and achieve the best fit

$$t := 0, .2 .. 20 \qquad\qquad i := 0 .. 8$$

$$P(t) := \frac{P_{Lsoln}}{1 - c_{soln} \cdot e^{-k_{soln} \cdot t}}$$

$$\text{Antdata} := \begin{pmatrix} 1 & 500 \\ 2 & 1000 \\ 3 & 2050 \\ 3.5 & 3800 \\ 4 & 5400 \\ 5 & 8700 \\ 6 & 10550 \\ 7 & 11200 \\ 8 & 11100 \\ 9 & 11900 \end{pmatrix}$$

observed data from plot above


Comparison of Model and Data
1.5·10^4
1·10^4
5000
size of colony
0
2
4
6
8
10
12
14
16
18
20
time in years
predicted ant population
measured data from plot by Gordon

Method 2 - use the "genfit" algorithm in Mathcad 5.0+

$$vt := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 3.5 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{pmatrix}$$

vector of time values

$$vant := \begin{pmatrix} 500 \\ 1000 \\ 2050 \\ 3800 \\ 5400 \\ 8700 \\ 10550 \\ 11200 \\ 11100 \\ 11900 \end{pmatrix}$$

vector of number of ants in colony corresponding to time

$$P = \frac{P_{Lsoln}}{1 - c_{soln} \cdot e^{-k_{soln} \cdot t}}$$

$$\frac{dP}{dk_{soln}} = \frac{-P_{Lsoln}}{\left(1 - c_{soln} \cdot \exp\left(-k_{soln} \cdot t\right)\right)^2} \cdot c_{soln} \cdot t \cdot \exp\left(-k_{soln} \cdot t\right)$$

$$\frac{dP}{dc_{soln}} = \frac{P_{Lsoln}}{\left(1 - c_{soln} \cdot \exp\left(-k_{soln} \cdot t\right)\right)^2} \cdot \exp\left(-k_{soln} \cdot t\right)$$

$$\frac{dP}{dP_{Lsoln}} = \frac{1}{\left(1 - c_{soln} \cdot \exp\left(-k_{soln} \cdot t\right)\right)}$$

$$F(t,u) := \begin{bmatrix} \dfrac{u_0}{1 - u_1 \cdot e^{-u_2 \cdot t}} \\ \dfrac{-u_0}{\left(1 - u_1 \cdot \exp(-u_2 \cdot t)\right)^2} \cdot u_1 \cdot t \cdot \exp(-u_2 \cdot t) \\ \dfrac{u_0}{\left(1 - u_1 \cdot \exp(-u_2 \cdot t)\right)^2} \cdot \exp(-u_2 \cdot t) \\ \dfrac{1}{\left(1 - u_1 \cdot \exp(-u_2 \cdot t)\right)} \end{bmatrix}$$

function to be fit

partial derivatives of function

$$u := \begin{pmatrix} 11100 \\ -5500 \\ 1.95 \end{pmatrix}$$

initial guesses for parameter values

$$P := \text{genfit}(vt, vant, u, F)$$

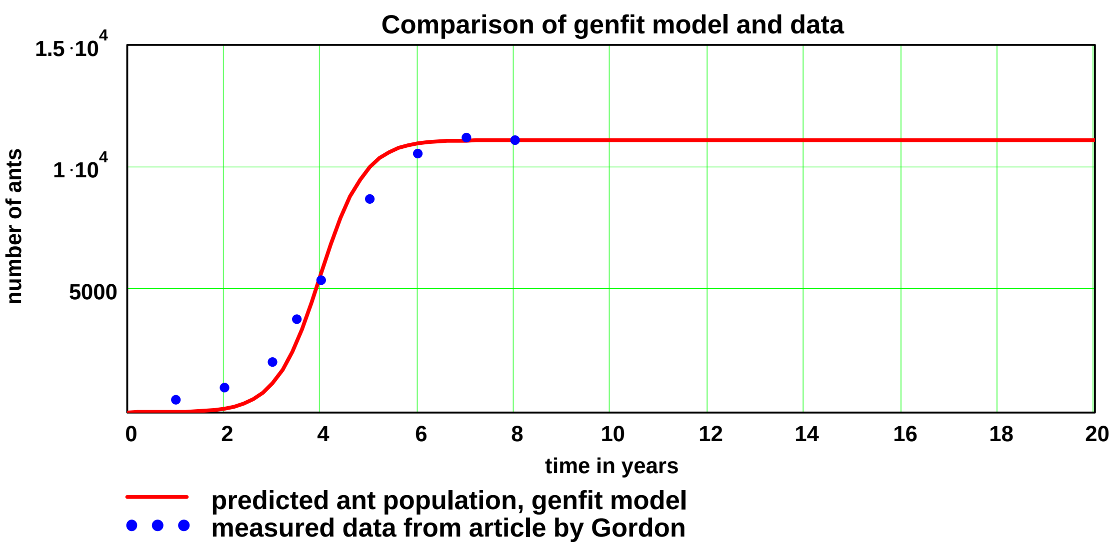
The Genfit algorithm computes values of the parameters (c,k and P_L here) which produce the best fit for the function chosen

$$P = \begin{pmatrix} 1.11 \times 10^{4} \\ -5.497 \times 10^{3} \\ 2.164 \end{pmatrix}$$

computed values of the parameters from genfit

$$f(t) := F(t, P)_0$$

equation for ant population using Genfit parameters



Now compare the two procedures :

$$P(t) := \frac{P_{Lsoln}}{1 - c_{soln} \cdot e^{-k_{soln} \cdot t}} \qquad t := 0, .2 .. 20$$