AN INTRODUCTION TO A MATHEMATICAL APPROACH TO THE STUDY OF KINSHIP

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Section 1: INTRODUCTION. Explication of the organization of behavior is as much a function of investigation as it is of organization itself. If there is organization, for example, but the investigation is inadequate because of imprecision or semantic ambiguity then the possibility of accurate explication is diminished. Therefore we have chosen mathematical language to investigate the organization of consanguineal relations.

Although this language focuses on a consanguineal-relation framework it does not imply a particular semantic theory of kinship terminology. A term such as 'father' can be described by reference to the consanguineal-relation framework as that kin type which is the male parent of ego. This does not imply that 'father' means "the male parent of ego". Just as one can describe 'neon' by reference to the framework of atomic structure without implying that its meaning is the atomic structure nor denying that its meaning may be a colorless, odorless, non-flammable gas, one can describe kinship terms by reference to a framework without implying that the meaning lies within that framework. The question of the meaning of a term is not at issue here.

We examine the organization of consanguineal kin as defined by a kinship terminology, utilizing certain aspects of Boolean algebra to concisely determine some formal characteristics of this organization. We wish to discover if these characteristics can provide useful insights for significant semantic or comprehensive theories of kinship.

A system of notation based upon the formal examination of the or-

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ganization of consanguineal relationships within kinship terminology is presented. Some of its advantages over previously used systems and some of its possibilities in furthering understanding of kinship are discussed.

Section 2: MATHEMATICAL BACKGROUND. We utilize the most basic aspects of the theory of Boolean algebras as realized by certain subsets of some set. Actually we deal with the Boolean algebra of all relations on a set noting that composition distributes over union. The composition $R_1 \circ R_2$ of two relations R_1 and R_2 is defined as follows: $R_1 \circ R_2 = \{(a,c) \mid \text{there exists } b \in X \text{ with } (a,b) \in R_1 \text{ and } (b,c) \in R_2\}$. There is a distinguished relation Δ on X, called the diagonal, defined by $\Delta = \{(x,x) \mid x \in X\}$ The reader is invited to verify that $R \circ \Delta = R$ and $\Delta \circ R = R$ for any relation R on X.

While the union and intersection operations are commutative, i.e., $A \cup B = B \cup A$ and $A \cap B = B \cap A$, the composition of relations is not. There exist relations R_1, R_2 such that $R_1 \circ R_2 \neq R_2 \circ R_1$. For example, let $R_1 = \{(1,2), (1,3)\}$ and $R_2 = \{(2,1)\}$, then $R_1 \circ R_2 = \{(1,1)\}$ and $R_2 \circ R_1 = \{(2,2), (2,3)\}$. A somewhat more subtle notion is that of associativity. Each of the above mentioned operations are associative, i.e., $(A \cup B) \cup C = A \cup (B \cup C)$, $(A \cap B) \cap C = A \cap (B \cap C)$ and $(R_1 \circ R_2) \circ R_3 = R_1 \circ (R_2 \circ R_3)$. Since the parentheses are irrelevant we shall drop them whenever possible. Note that $(R_1 \circ R_2) \cup (R_1 \circ R_3) = R_1 \circ (R_2 \cup R_3)$ and $(R_2 \circ R_1) \cup (R_3 \circ R_1) = (R_2 \cup R_3) \circ R_1$. Letting $R_2 = \Delta$, we have $R_1 \cup (R_1 \circ R_3) = R_1 \circ (\Delta \cup R_3)$. Similarly, $R_1 \cup (R_3 \circ R_1) = (\Delta \cup R_3) \circ R_1$.

One may also define

$A_1 \cup A_2 \cup \ldots \cup A_n = \{x \mid x \in A_1 \text{ or } x \in A_2 \text{ or } \ldots \text{ or } x \in A_n\}$,
$A_1 \cap A_2 \cap \ldots \cap A_n = \{x \mid x \in A_1 \text{ and } x \in A_2 \text{ and } \ldots \text{ and } x \in A_n\}$, and
$R_1 \circ R_2 \circ \ldots \circ R_n = \{(a,b) \mid \text{there exists } a_0, a_1, \ldots, a_n \text{ with } (a_{i-1},a_i) \in R_i, \text{ for } i = 1, 2, \ldots, n, \text{ and } a_0 = a, a_n = b\}$.

Let $R^{(n)} = R \circ R \circ \ldots \circ R$ n-times. (We reserve the usual symbol R^n for something else.) By convention $R^{(0)} = \Delta$. Associativity guarantees that these definitions agree with the old ones, for example $((R_1 \circ R_2) \circ (R_3 \circ R_4)) \circ R_5 = R_1 \circ R_2 \circ R_3 \circ R_4 \circ R_5$. As long as the order of the R_i's is maintained any meaningful concatenation of parentheses gives the same result; thus we may as well drop the parentheses and work with $R_1 \circ R_2 \circ \ldots \circ R_n$.

The reason for belaboring this point is that the operation which we shall utilize the most is not associative. Following Atkins[1] [1974], we define it now, leaving the motivation for later. Let n be an integer with $n \geq 1$, and let $R_1, \ldots, R_n$ be relations. We define the product $R_1R_2 \ldots R_n$ as follows:

$R_1R_2 \ldots R_n = \{(a,b) \mid$ there exist $a_0, a_1, \ldots, a_n$ such that $(a_{i-1}, a_i) \in R_i$, for $i = 1, \ldots, n$, $a_0 = a$, $a_n = b$, and if $a_i \neq a_j$ then $a_i \neq a_k$ where $1 \leq i < j \leq k \leq n\}$, let $R^n = RR \ldots R$ n-times.

Note that $R_1R_2 \ldots R_n$ is precisely the set of elements $(a,b) \in R_1 \circ R_2 \circ \ldots \circ R_n$ for which there is a sequence $a_0, a_1, \ldots, a_n$ linking a and b with $a_i \neq a_j$ if $i < j$ and $a_i \neq a_{i+1}$. $R_1R_2 \ldots R_n$ is called the geneaproduct of the relations $R_1, R_2, \ldots, R_n$.

To show that associativity may fail (even if $n = 2$) we give an example of three relations R_1, R_2, R_3 such that $(R_1R_2)R_3 \neq R_1(R_2R_3)$. Let $R_1 = \{(1,2)\}$, $R_2 = \{(2,3), (2,1)\}$ and $R_3 = \{(3,4), (1,5)\}$. Then $(R_1R_2)R_3 = \{(1,4)\}$ whereas $R_1(R_2R_3) = \{(1,4), (1,5)\}$.

Note that $R_1R_2 \cup R_1R_3 = R_1(R_2 \cup R_3)$, and $R_2R_1 \cup R_3R_1 = (R_2 \cup R_3)R_1$. Note that $R\Delta = \Delta R = R$ and $R_1R_2 \ldots R_n = R_1R_2 \ldots R_i \Delta R_{i+1} R_{i+2} \ldots R_n$ so that $R_1 \cup R_1R_3 = R_1(\Delta \cup R_3)$ and $R_1 \cup R_3R_1 = (\Delta \cup R_3)R_1$.

Let J denote the diversity relation: $J = \{(x,y) \mid x \neq y\}$. As Atkins points out, the effect of the clause "if $a_i \neq a_j$, then $a_i \neq a_k$ where $1 \leq i < j \leq k \leq n$" in the definitions of $R_1R_2 \ldots R_n$ is, "That J, the normally nontransitive diversity relation, is made transitive within all relational strings employing the geneaproduct operator. What this

[1] A slightly different operation was defined in an earlier version of the paper (Greechie and Ottenheimer, 1972). We follow Atkins for reasons of parsimony.

stipulation assures is that the geneatracings involving 'doubling back' are excluded as empirical interpretations of geneaproduct strings."

We conclude this section with two definitions. Let R be a relation, define $R^{-1} = \{(b,a) | (a,b) \in R\}$. Note that $(R^{-1})^{-1} = R$. Let A be any set. A family $A_1, A_2, \ldots, A_n$ of non-empty subsets of A is a finite <u>partition</u> of A in case $A_1 \cup A_2 \cup \ldots \cup A_n = A$ and $A_i \cap A_j = \phi$ whenever $i \neq j$.

<u>Lemma 2.1</u>.

(1) $(R^{(n)})^{-1} = (R^{-1})^{(n)}$; (2) $(A \times B)^{-1} = B \times A$; (3) $(R_1 \cup R_2)^{-1} = R_1^{-1} \cup R_2^{-1}$; (4) $(R_1 \cap R_2)^{-1} = R_1^{-1} \cap R_2^{-1}$; (5) $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$; (6) $(R_1R_2)^{-1} = R_2^{-1}R_1^{-1}$; (7) If $A \subseteq B$ then $A = A \cap B$ and $B = A \cup B$; (8) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

These facts are immediate consequences of the above definitions.

Section 3: <u>NOTATION FOR THE ANALYSIS OF KINSHIP TERMINOLOGY</u>. Let S be the set of all individuals which make up a society. We are interested in relations between the individuals of the society, i.e., in subsets $R \subseteq S \times S$. More specifically we are interested in certain relations derived from the relation P where $P = \{(a,b) | a \text{ is a parent of } b\}$. Let $\underline{\underline{C}} = \{(a,b) | a \text{ and } b \text{ are consanguineally related}\}$. (All seemingly homeless individuals live in S, i.e., a, b $\in$ S unless otherwise specified.) (We regard each a $\in$ S as an ancestor of himself.) We shall now express $\underline{\underline{C}}$ explicitly in terms of P and P^{-1}:

$$(3.1) \qquad \underline{\underline{C}} = \bigcup_{i,j=0}^{\infty} (P^{-1})^{(i)} \circ P^{(j)}.$$

3.1 is obtained by a simple argument. $(a,b) \in \underline{\underline{C}}$ if and only if a and b have a common ancestor c, i.e., if and only if $(c,a) \in P^{(i)}$ and $(c,b) \in P^{(j)}$ for some non-negative integers i and j. (Note that if, say, $i = 0$, then $c = a$ and $(a,b \in P^{(j)}$.) Thus $(a,b) \in \underline{\underline{C}}$ if and only if, for some c, $(a,c) \in (P^{(i)})^{-1} = P^{-1})^{(i)}$ and $(c,b) \in P^{(j)}$, that is, if and only if $(a,b) \in (P^{-1})^{(i)} \circ P^{(j)}$.

We now turn our attention to the consanguineal relations and kinship terminology. We group together those individuals who, relative to some fixed but arbitrary individual i_o are called the same term by i_o. All individuals disappear from the presentation and only the relations are indicated. We begin by distinguishing the sexes. Let S_1 be the set of all males in S and let S_2 be the set of all females in S. Then $S = S_1 \cup S_2$ and $S_1 \cap S_2 = \emptyset$. This partition of S induces a partition of $S \times S$ into four relations: $S_1 \times S_1$; $S_1 \times S_2$; $S_2 \times S_1$; and $S_2 \times S_2$.

Fig. 1.

	S_1	S_2
S_1	$S_1 \times S_1$	$S_1 \times S_2$
S_2	$S_2 \times S_1$	$S_2 \times S_2$

The latter partition is denoted diagrammatically in Figure 1. It is convenient to consider the four "quadrants" two at a time. We therefore define the following relations.

<u>Definition 3.2</u>. $X_1 = (S_1 \times S_1) \cup (S_2 \times S_2)$; $X_2 = (S_1 \times S_2) \cup (S_2 \times S_1)$; $X_3 = (S_1 \times S_1) \cup (S_2 \times S_1)$; $X_4 = (S_1 \times S_2) \cup (S_2 \times S_2)$; $X_5 = (S_1 \times S_1) \cup (S_1 \times S_2) \cup (S_1 \times S_2)$; $X_6 = (S_2 \times S_1) \cup (S_2 \times S_2)$.

<u>Remark</u>. $X_1^{-1} = X_1$, $X_2^{-1} = X_2$, $X_3^{-1} = X_5$, $X_4^{-1} = X_6$, $X_5^{-1} = X_3$, $X_6^{-1} = X_4$.

Note that X_1 and X_2 are the "diagonals", X_3 and X_4 are the "columns", and X_5 and X_6 are the "rows" of Figure 1. Also note that we have exhausted all possible two-element subsets of the set $\{S_1 \times S_1, S_1 \times S_2, S_2 \times S_1, S_2 \times S_2\}$. Free, for our trouble, we obtain three more partitions of $S \times S$:

(*) $$S \times S = X_1 \cup X_2 = X_3 \cup X_4 = X_5 \cup X_6$$

($X_i \cap X_{i+1} = \emptyset$, for i odd).

We use these partitions to partition any relation $R \subseteq S \times S$ by defining $R_i = R \cap X_i$. Thus, with $R = P$, we have $P_i = P \cap X_i$; and therefore

(**) $$P = P_1 \cup P_2 = P_3 \cup P_4 = P_5 \cup P_6.$$

The argument proving these equalities is simple. For example, we prove that $P = P_1 \cup P_2$ as follows: Since $P \subseteq S \times S$, $P = P \cap (S \times S) = P \cap (X_1 \cup X_2) = (P \cap X_1) \cup (P \cap X_2) = P_1 \cup P_2$. Use part (8) of Lemma 2.1 to justify the third equality.)

Let $C = P^{-1}$ so that $C_i = C \cap X_i = P^{-1} \cap X_i$. Noting that P_i^{-1} stands for $(P_i)^{-1}$, we have the following: $P_1^{-1} = C_1$, $P_2^{-1} = C_2$, $P_3^{-1} = C_5$, $P_4^{-1} = C_6$, $P_5^{-1} = C_3$, $P_6^{-1} = C_4$.

Also (***) $C = C_1 \cup C_2 = C_3 \cup C_4 = C_5 \cup C_6$.

To illustrate the ease with which any of the above equations may be derived, we prove that $P_3^{-1} = C_5$: $P_3^{-1} = (P \cap X_3)^{-1} = P^{-1} \cap X_3^{-1} = C \cap X_5 = C_5$.

We now have the basic elements for expressing the consanguineal relationships in kinship terminology:

1. $(a,b) \in P$ means a is the parent of b,
2. $(a,b) \in P_1$ means a is the parent of b and they are of the same sex,
3. $(a,b) \in P_2$ means a is the parent of b and they are of the opposite sex,
4. $(a,b) \in P_3$ means a is the parent of b and b is male,
5. $(a,b) \in P_4$ means a is the parent of b and b is female,
6. $(a,b) \in P_5$ means a is the parent of b and a is male,
7. $(a,b) \in P_6$ means a is the parent of b and a is female,
8. $(a,b) \in C$ means a is the child of b,
9. $(a,b) \in C_1$ means a is the child of b and they are of the same sex,
10. $(a,b) \in C_2$ means a is the child of b and they are of the opposite sex,
11. $(a,b) \in C_3$ means a is the child of b and b is male,
12. $(a,b) \in C_4$ means a is the child of b and b is female,
13. $(a,b) \in C_5$ means a is the child of b and a is male,
14. $(a,b) \in C_6$ means a is the child of b and a is female.

One might be tempted to replace, for example, P_5 or C_3 with the traditional kinship notation symbol 'F' since $(a,b) \in P_5$ means a is the male parent of b and $(a,b) \in C_3$ means a is the child of the male parent b. We do not do this basically because of the widespread use of the symbol F to denote an individual or kin type. aF substituted for $(a,b) \in C_3$ may inadvertently be understood as "a's father" rather than "a is the child of". We wish to keep clear that what we are dealing with is the parental relationship between individuals and to avoid confusing the relationship with any element involved in the relationship.[1]

We now present an example using the notation. Table I displays the expressions of the consanguineal relationships for 13 English kin terms. The last column contains the expressions of these consanguineal relationships simplified by repeatedly utilizing the distributivity of composition over unions (cf. (1*)). This permits simplifying the class of relationships in the framework that fall within the scope of 'aunt', for example, as follows:

(3.2) $$C_3C_3P_4 \cup C_3C_4P_4 \cup C_4C_3P_4 \cup C_4C_4P_4 = C_3(C_3P_4 \cup C_4P_4) \cup C_4(C_3P_4 \cup C_4P_4) = (C_3 \cup C_4)(C_3P_4 \cup C_4P_4) = C(C_3P_4 \cup C_4P_4) = C(C_3 \cup C_4)P_4 = CCP_4.$$

Thus in place of (3.2) we need only write CCP_4.

The elements and their sex-denoting subscripts are concatenated in the form $C^iP^j (i,j \geq 0)$ utilizing the non-regressive geneaproduct. The reasons for, and the significance of, this form and the geneaproduct in the analyses of kinship terminology will now be made explicit.

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[1] It is possible, however, in the above notation to deal with the individuals involved in any relationship. For example, $aP_5 = \{b \mid (a,b) \in P_5\}$ is the set of children of a. In what follows we shall avoid expressions such as aP_5 and use simply P_5 with ego assumed to precede the expression. We do not concern ourselves with the problem of whether the term 'child' semantically denotes aP_5 rather than P_5. For us the two possibilities are interchangeable. We choose to work with the latter— ignoring the semantics— because it is more abstract.

TABLE I: ENGLISH, MALE OR FEMALE EGO

Kin Term	Traditional Notation	Relationships	Simplified Boolian Expression
Grandfather	FF;MF	C_3C_3; C_4C_3	CC_3
Grandmother	FM;MM	C_3C_4; C_4C_4	CC_4
Father	F	C_3	C_3
Mother	M	C_4	C_4
Uncle	FB[1]	$C_3C_3P_3$, $C_3C_4P_3$;	CCP_3
	MB	$C_4C_3P_3$, $C_4C_4P_3$	
Aunt	FZ	$C_3C_3P_4$, $C_3C_4P_4$;	CCP_4
	MZ	$C_4C_3P_4$, $C_4C_4P_4$	
Brother	B	C_3P_3, C_4P_3	CP_3
Sister	Z	C_3P_4, C_4P_4	CP_4
Cousin[2]	FBS	$C_3C_3P_3P_3$, $C_3C_4P_3P_3$, $C_3C_3P_4P_3$, $C_3C_4P_4P_3$, $C_4C_3P_3P_3$, $C_4C_4P_3P_3$;	CCPP
	MZS	$C_4C_3P_4P_3$, $C_4C_4P_4P_3$;	
	FBD	$C_3C_3P_3P_4$, $C_3C_4P_3P_4$;	
	FZD	$C_3C_3P_4P_4$, $C_3C_4P_4P_4$;	
	MBD	$C_4C_3P_3P_4$, $C_4C_4P_3P_4$;	
	MZD	$C_4C_3P_4P_4$, $C_4C_4P_4P_4$	
Son	S	P_3	P_3
Daughter	D	P_4	P_4
Grandson	SS;DS	P_3P_3; P_4P_3	PP_3
Granddaughter	SD;DD	P_3P_4; P_4P_4	PP_4

[1] In the present notation, brother (B) is a derived concept: $B = C_3P_3 \cup C_4P_3$. Similarly, sister (Z) is $C_3P_4 \cup C_4P_4$.

[2] We have arbitrarily truncated the expansion for the term 'cousin' at the first-cousin level. Alternative variations clearly fit into the same framework.

Section 4: DISCUSSION. To illustrate some of the significance of the geneaproduct for analyzing kinship data we begin by examining in detail an example from the Comoro Islands. Table II displays from Domoni, a community in the islands, a set of kin terms, the respective kin type denotata for the terms (utilizing traditional notation), and the respective consanguineal relationships involved utilizing the notation developed above.[1]

TABLE II: COMORO, MALE OR FEMALE EGO

Kin Term	Traditional Notation	Simplified Boolian Expression
mbakoko	FF, MF	CC_3
koko	MM, FM	CC_4
mbaba	F, FB, MB	$C_3 \cup CCP_3$
mmama	M, MZ, FZ	$C_4 \cup CCP_4$
mwananya	B, Z, MBS, FBS, MZS, FZS, MBD, FBD, MZD, FZD	$C(\Delta \cup CP)P$
mwana	BD, ZD, S, D, BS, ZS	$(\Delta \cup CP)P$
mjuhu	SS, SD, DS, DD	PP

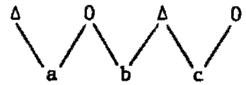
The set of terms corresponds to the "Scale type 5" of Buchler's (1964) analysis of Hawaiian-type kinship terminologies. Buchler, following Lounsbury (1964) provides a formal account with a set of rules for extending or reducing the kin types denoted by a term or to a "focal type". But careful examination proves that the rules specified do not provide an adequate formal analysis of the system. The corollary to the stronger form II of the merging rule, for example, states "MB... → F...". This is to be read as "Let the kin type Mother's Brother, when-

[1] Both the number of the terms and the range of each entry have been restricted here. For more complete discussion see Ottenheimer (1971).

ever it occurs as link between ego and any other relative, be regarded as equivalent to the kin type Father, in the context" (Buchler, 1964: 291). This rule permits transformations of the type MBS → FS which reduces to B with the use of the "half-sibling rule" (FS → B). Examination of the data shows that in this case MBS and B do fall within the range of the same term. But if we utilize the merging rules as an expansion rule we see that it produces MBF from FF, for example, and MBF is not an entry in the table and, furthermore, in Domoni the kin type MBF may not be terminologically equivalent to FF nor even consanguineally related to ego. If M and MB are half-siblings through a common mother with different fathers then it is obvious that MBF and MF are not the same. Thus, it is possible that ego is not necessarily related to MBF and consequently, in Domoni, will not refer to or address MBF by the term for MF or FF. This points to two interesting phenomena with regard to the organization of relationships in kinship. The first is formal relationships between the organization of consanguineal relationships of terminology and marriage rules. We will return to this in a moment. Let us briefly point out now that the rules of an adequate formal account of kinship terminology are not independent of the type of marriage rules of the system. In Domoni, where there is polygyny, serial monogamy and, consequently, numbers of half-siblings, Buchler's rules do not provide an adequate analysis and other rules must be specified.

To define formally, at least in part, the nature of this relationship between the rules of terminology and marriage rules would be of great interest to anthropologists. This becomes possible with the system outlined above. The second point of interest is related to this possibility. This point is the formal reason for the failure of Buchler's analysis. It rests upon a fundamental distinction between the relationships involved with half-siblings and full-siblings. These relationships are not equivalent. If we let R_1 and R_2 denote the full-sibling relationship and the half-sibling relationship respectively, then $R_1 = R_2$ only if both relationships are transitive. Any relation, R, is transitive if the following condition necessarily holds: For any elements _a_, _b_, _c_, if $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$. R_1 is clearly transitive. If _a_ is a full sibling of _b_ and _b_ is the full-sibling of _c_ [$(a,b) \in R$, and $(b,c) \in R$)], then it follows that _a_ is the full sibling of _c_ [$(a,c) \in R$]. But the relation R_2 is non-transitive. This may be seen by examining Figure 2. Here _a_ is the half-sibling of _b_ and _b_ is the half-sibling of _c_ [$(a,b) \in R_2$ and $(b,c) \in R_2$], but _a_ is not the half-sibling of _c_ (not aR_2c). Thus, $R_1 \neq R_2$.

Fig. 2.



If we examine the half-sibling rule as it is utilized in Buchler's analysis of Hawaiian-type systems (and many other formal analyses as well) we see essentially two problems. First is the problem of ambiguity. Looking at _FS_ → _B_ as an example of the half-sibling rule, we see that in terms of consanguineal relations there are four distinct possible interpretations of _B_: 1) C_3P_3 2) C_4P_3 3) $C_3P_3 \cup C_4P_3$ 4) $C_3P_3 \cap C_4P_3$. Now if the ambiguity is avoided by specifying one of the interpretations, other problems can arise. If C_3P_3 is selected then the half-sibling rule becomes the identity $C_3P_3 = C_3P_3$ (_FS_ → _FS_) and it is obviously of little use as an expansion or reduction rule. If either the second, third or fourth interpretation is selected then the fact that the half- and full-sibling relationships are not equivalent means that the half-sibling rule can be applied as an operator of consanguineal relationships if and only if full-siblings are involved. In interpreting _B_ as $C_3P_3 \cap C_4P_3$ (full-brother), the half-sibling rule ($C_3P_3 = C_3P_3 \cap C_4P_3$) will obviously be violated in any case that $C_3P_3 \neq C_4P_3$ viz., any case in which ego has a half-brother. Likewise, the same is true for the other forms of the half-sibling rule (_FD_ → _Z_, _MS_ → _B_, _MD_ → _Z_).

The advantage in utilizing the geneaproduct is that it permits a precise and abstract display of the relationships of consanguinity. This, in turn, helps in the reduction of ambiguity in the tools used for the analysis of kinship and also permits the abstract manipulation which can point to the characteristics of the data which are not immediately apparent. Another advantage, in regard to formal accounts of kinship terminology, is that it permits the construction of a set of rules that adequately accounts for the data.

A formal account of the Comorian kinship presented in Table II requires only three rules: $C_i \equiv CCP_i$ $(i = 3,4)$, $CP \equiv CCPP$, and $P \equiv CPP$. This permits the accurate expansion or reduction of all the expressions from or to the focal expressions:[1] CC_3, CC_4, C_3, C_4, CP, P, PP. Furthermore, it does not utilize a half-sibling rule and, consequently, no assumption that only full-siblings are involved. Thus it is more faithful to the data.

Application of the geneaproduct to other systems of kinship terminology has revealed inadequacies in other analyses and the possibility that the half-sibling rule is not necessary for an adequate formal account of any kinship terminological system. This suggests, furthermore, that the use of the sibling link as primary in the analysis of kinship is not required.

The non-regressive geneaproduct with the concatenation of the parental relation and its converse in the form C^iP^j and the subscripts denoting sex permits adequate analysis of the consanguineal relationships involved in kinship terminological systems. It also provides for efficiency in the expression of the consanguineal relationship. Thus, the Comorian term "mwananya" can be expressed simply as $CP \cup CCPP$ as opposed to the requirements of traditional notation which necessitate the list: <u>B</u>, <u>Z</u>, <u>MBS</u>, <u>FBS</u>, <u>MZS</u>, <u>FZS</u>, <u>MBD</u>, <u>FBD</u>, <u>MZD</u>, <u>FZD</u>.

Of course, this means of approaching kinship terminology does not attempt to say everything about this highly complex and varied subject. It cannot. It is restricted primarily to the relationships of consanguinity which provide a framework for only a segment of the relationships and elements of the universe of discourse that is recognizable as proper to kinship terminology. But why assume that a method must take into account all aspects of a subject under question? Would it not be sufficient if the method provides certain insights into the sub-

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[1] A more detailed discussion of the application of these rules to obtain any consanguineal relation involves the notion of instantiation and will not be treated here.

ject and provides a basis for significant contributions for at least a part of the area? That this approach can provide insights we believe we have demonstrated above. It is to the question of contribution we now turn.

Someone may argue that the inaccuracies of the formal accounts noted above can be dealt with essentially by omitting B and Z from analyses, utilizing only FS, MD, etc. instead and rewriting the rules to accurately account for the data. If this is done, however, aside from losing some of the abstractness of the present notation one may overlook the non-regressive nature of the system. Thus, if "brother" were to be denoted as FS and MS this would be incorrect if FS and MS would include ego as well as C_3P_3 and C_4P_4. Ego is not a member of the class denoted as "brother". Likewise for FFS if it does not distinguish between C_3 and $C_3C_3P_3$. Thus, the non-regressive nature of the notation provides an explicit means for analyzing the data properly that is otherwise dealt with implicitly, if at all.

The specific concatenation of the elements, viz., C^iP^j, provides the means for the preventing the generating of unwanted relationships or producing kin types that are not consanguineals of ego. The expression CPCP, for example, does not occur because for any a and b such that aCPCPb,b is not necessarily a consanguineal relative of a. That is, if CP is interpreted as sibling, it does not follow that a's sibling's sibling is a sibling of a. This follows from the non-transitivity of the half-sibling relationship and the realization that sibling can mean either full- or half-sibling. One notes, however, that it will follow that expressions of the form P^jC^i can be permitted in an expression if cousin marriage is the rule. Thus, for example, the expression CPCP will represent a consanguineal relationship where ego's half-sibling's half-sibling is the child of ego's parent's cousin. Also, the expression CPC denotes the relationship ego's parent's cousin but not necessarily ego's parent. If strict monogamy is the rule then CPC = C. In strict monogamy a man has children with only one woman and vice versa. Any siblings are the offspring of the same father and mother, that is, all siblings are full-siblings. Then the expression CPC denotes ego's parent. Also, the expression PC will denote ego's spouse. Thus, the concatenation of elements can be utilized to denote not only the organization of relationships of kinship terminology but also certain marriage relationships. With the abstract nature of the geneaproduct the possible relationships between areas of terminology and marriage become more evident.

The non-associativity of the geneaproduct also has implications for investigation of marriage relationships. Any attempt to analyze marriage utilizing models that assume associative operations cannot be expected to adequately account for the data if the marriage system is based upon terminological relationships. Thus, in the examination of marriage from the point of view of classes, a model utilizing associative operations may not be adequate if the classes are defined terminologically. Possible applications along the lines indicated by Weil (1963), Bush (1963), White (1963) and, most recently, Boyd (1969) should be carefully undertaken.

The advantage of utilizing consanguineal relationships as the focus of analysis rather than consanguineal kin types should now be obvious. Marriage is a relationship and it becomes easier to analyze this relationship with regard to other areas of behavior if the other areas are treated on the same level. Thus, the geneaproduct permits not only a precise and abstract means for examining kinship terminology and certain marriage relationships but, also, a means for examining the formal relationships between the two.

Other areas of behavior can also be included. Descent, of course, immediately suggests itself. We can also treat inheritance and succession, for example, as a system of relationships which can be examined and compared with the above when the framework of consanguineal relationships applies. Since in most societies both property and offices are matters of consanguineal concern or, at least, can be examined with regard to the consanguineal framework, the use of the geneaproduct to these areas should be widely applicable. In this regard a specific strategy suggests itself. Since Rivers (1914) defined inheritance and succession as the transmission of property and the transmission of office respectively, it has been the practice to examine them independently and then attempt to find some grounds upon which to compare them. If, however, we shift the emphasis and treat inheritance as the transmission of people with regard to property and succession as the transmission of people with regard to office then we immediately see the common ground for their comparison. We can treat property in the same manner we treat terminology. Instead of looking for kin terms, however, we look for "property terms"— the significant properties within the culture— and stipulate the "property relationships" that are associated with the various properties. In Domoni again, for example, the relationships between the potential inheritors of a house and ego as owner of the house is described as $C_4 \cup P_4 \cup CPP_4$. For cattle the expression is $C_3 \cup P_3 \cup CPP_3$. Thus house, cattle, etc. become "property terms" that define various classes of relationships. When expressed in terms of the geneaproduct these are amenable to formal comparison and on a level comparable to those expressions for kinship terminology. Similarly for succession. We can then examine the relationship of the kin terms, the property terms, the succession terms, etc. and compare them with reference to the geneaproduct to determine the formal relationships between them.[1]

Aside from the comparison of different aspects of behavior within a culture the geneaproduct also permits the formal examination of areas between cultures. In the Appendix is an example of comparison between cultures. It is a mathematical treatment of the relationship between Crow- and Omaha-type I terminologies. This provides a concise and formal statement of the "mirror-image" relationship between these two terminologies.

In conclusion, two points should be reiterated. First, the use of the geneaproduct for the examination of the organization of relationships within and between cultures does not imply a specific theory or

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[1] For a provocative essay toward this type of approach see Reining (1972).

meaning. It is neither a statement for or against either the extensionist or the sociological view of kinship terminology. Above we have touched upon a formal account of kinship terminology to demonstrate the potential of the geneaproduct. Specifically for this type of analysis the geneaproduct does permit more accurate analyses. More important, in our view, the geneaproduct provides a means for accurate and abstract examination of the organization of a number of social relationships. Second, the above does not intend to be an attempt to answer all of the problems even in the one area of kinship terminology. The geneaproduct is meant to apply primarily only to those areas where a genealogical framework is appropriate. But since consanguinity and affinity play a part in or can be applied to all cultures and a large part in some cultures, it is not a minor undertaking.

If it is questioned whether or not human behavior is organized and predictable then any attempt to resolve the question requires utilization of precise techniques. For if unwanted ambiguity and inaccuracy are a part of the investigator's research tools then any conclusion that arises is suspect. A denial of organization and human predictability, for example, may simply be a result of inadequate tools and not a characteristic of the data at all. If, on the other hand, it is understood that human behavior is organized, at least in part, then an approach that permits exposition of at least part of that organization is useful. The consanguineal relation framework does not in itself explain why behavior is organized the way it is nor does it presuppose an ontological status of the principles of organization—whether they are in the mind of the native or the investigator. It simply permits a formal means for the investigation and comparison of the organization of behavior in areas within any culture and between all cultures in a formal manner. It minimizes ambiguity, promotes accuracy, displays in an effective manner important relationships, and may lead to a better understanding of how (not why) human behavior is organized. This is where its significance lies.

APPENDIX: THE COPENHAGEN FUNCTION

In this appendix we apply the mathematical notation introduced earlier to the phenomenon of "mirror image" societies. We show that the terminology for the system Crow-type I may be obtained from the terminology for the system Omaha-type I and vice-versa. The connection is effected by a mapping which we call the Copenhagen function which maps terms of one society to terms of the other. Loosely speaking, the Copenhagen function changes the sex of everyone in the society. Somewhat more precisely, it maps the term for a relation such as "a male's father's mother's daughter in one society to the term for a female's mother's father's son" in the second society. The main advantage to our approach is its precision and the suggestions which it makes for ordering and comparing the terminology of diverse societies. The mathematically naive reader is referred to Foulis (1969) for elementary definitions.

Let $\mathcal{C} = \{C_{n_1} \circ C_{n_2} \circ \ldots \circ C_{n_i} \circ P_{m_1} \circ P_{m_2} \circ \ldots \circ P_{m_j} \mid$ each $C_{n_k} \in \{C, C_3, C_4\}$, each $P_{m_k} \in \{P, P_3, P_4\}$ $0 \leq i, j\}$ and let $\underline{\underline{C}} = \cup\, \mathcal{C}$. Recall that the set of all relations on S is denoted by $\mathcal{R}(S)$ so that $\mathcal{R}(S) = \{R \mid R \subseteq S \times S\}$.

Define $\beta : \mathcal{C} \to \mathcal{R}(S)$ by $\beta(C_{n_1} \circ C_{n_2} \circ \ldots \circ P_{m_j}) = C_{n_1} C_{n_2} \ldots P_{m_j}$. Let $\mathcal{C}_0$ denote the set of all finite unions of relations in the image of β so that $\mathcal{C}_0 = \{R_1 \cup R_2 \cup \ldots \cup R_n \mid$ for each $i = 1, 2, \ldots, n$ there exists $R_i' \in \mathcal{C}$ with $R_i \in \beta(R_i')\}$. Let $\underline{\underline{C}}_0 = \cup\, \mathcal{C}_0$. It is easy to see that $\underline{\underline{C}} = \underline{\underline{C}}_0$.

Define $\mu : \mathcal{C}_0 \to \mathcal{R}(S)$ and $\phi : \mathcal{C}_0 \to \mathcal{R}(S)$ by $\mu(R) = R \cap (S_1 \times S)$, $\phi(R) = R \cap (S_2 \times S)$. Let $\mathcal{C}_{\mu} = \{\mu(R) \mid R \in \mathcal{C}_0\}$ and $\mathcal{C}_{\phi} = \{\phi(R) \mid R \in \mathcal{C}_0\}$ be the images of μ and ϕ respectively.

Let $\mathcal{T}_\mu$ be a partition of a subset $\mathcal{D}_\mu$ of $\mathcal{C}_\mu$, let $T \in \mathcal{D}_\mu$ and let $[T]_\mu$ denote the cell of the partition which contains T. $\mathcal{T}_\mu$ is called a $\underline{\mu}$-terminology in case the following two properties hold: (1) If $R, S \in [T]_\mu$, then $R \cup S \in [T]_\mu$ and (2) if $R \subseteq S$, $R \in \mathcal{D}_\mu$ and $S \in [T]_\mu$, then $R \in [T]_\mu$, then cells of the partition $\mathcal{T}_\mu$ are called $\underline{\mu}$-terms. A $\underline{\phi}$-terminology $\mathcal{T}_\phi$ and $\underline{\phi}$-terms are defined similarly; a typical ϕ-term is denoted $[R]_\phi$ (where $R \in \mathcal{D}_\phi$).

Let T_μ be a μ-terminology, T_ϕ a φ-terminology, set $T = T_\mu \cup T_\phi$ and call T a *terminology* (for the society S). Notice that $T_\mu \cap T_\phi = \Phi$, in fact $(\cup T_\mu) \cap (\cup T_\phi) = \Phi$.

Let $\sigma[R]_\mu \underset{d.f.}{=} \cup [R]_\mu$ so that $\sigma([R]_\mu) = \{(a,b) \mid (a,b) \in Q$ for some $Q \in [R]_\mu\}$. Similarly let $\sigma([R]\ ) \underset{d.f.}{=} \cup [R]_\phi$. Note that $\{\sigma([R]_\mu) \mid [R]_\mu \in T_\mu\}$ is *not* a partition of $S_1 \times S$. This mirrors the fact that it is possible for a pair (a,b) to be simultaneously a member of two inequivalent relations; thus a may call b by two distinct kin-terms.

Let S and S' be societies with consanguineal relations C and C' and terminologies $T = T_\mu \cup T_\phi$ and $T' = T'_\mu \cup T'_\phi$, respectively. A mapping $f: C \to C'$ is said *to respect the terminologies* if $f([R]) \in T'$ for all $[R] \in T$. (Recall that $f([R]) = \{f(S) \mid S \in [R]\}$.) Such a mapping f induces a mapping $\tilde{f}: T \to T'$ defined, for each $[R] \in T$, by $\tilde{f}([R]) = f([R])$; $\tilde{f}$ is called a *terminological mapping*. Let $\tilde{f}$ be a terminological mapping. Then $\tilde{f}$ is called *Type 1* if $f|T_\mu: T_\mu \to T'_\mu$ and $f|T_\phi: T_\phi \to T'_\phi$. $\tilde{f}$ is called a *Type 2* if $\tilde{f}|T_\mu: T_\mu \to T'_\phi$ and $\tilde{f}|T_\phi: T_\phi \to T'_\mu$. A terminological mapping $\tilde{f}$ which is a bijection and Type 1 (respectively, Type 2) is called a terminological isomorphism (respectively, a *terminological dual isomorphism*). If such an $\tilde{f}$ exists, then T and T', S and S', are called terminologically isomorphic (respectively *terminologically dually isomorphic*).

Let S and S' be societies. Then $S = S_1 \cup S_2$ and $S' = S'_1 \cup S'_2$, where S_1 is the set of males in S and S_2 is the set of females in S; S'_1 and S'_2 are the corresponding sets in S'. In general, in a set or a relation, a prime indicates that that set or relation appears in the society S' rather than in the society S. Let c_0 be the mapping from $\{S_1,S_2\}$ to $\{S'_1,S'_2\}$ defined as follows: $c_0:\{S_1,S_2\} \to \{S'_1,S'_2\}$, $c_0(S_1) = S'_2$, $c_0(S_2) = S'_1$. Then c_0 induces a mapping c_1 from the set $\{S_1 \times S_1, S_1 \times S_2, S_2 \times S_1, S_2 \times S_2\}$ to the set $\{S'_1 \times S'_1, S'_1 \times S'_2, S'_2 \times S'_1, S'_2 \times S'_2\}$ defined as follows: for $i,j \in \{1,2\}$, $c_1(S_i \times S_j) = c_0(S_i \times c_0(S_j)$.

Then c_1, in turn, induces a function c_2 from the set $\{X_i \mid i = 1, 2, \ldots, 6\}$ to the set $\{X'_i \mid i = 1, 2, \ldots, 6\}$ (see 3.2 for the definition of X_i) defined as follows: $c_2(X_1) = X'_1$, $c_2(X_2) = X'_2$, $c_2(X_3) = X'_4$, $c_2(X_4) = X'_3$,

$c_2(X_5) = X_6'$, $c_2(X_6) = X_5'$. For example, $c_2(X_3) = c_2((S_1 \times S_1) \cup (S_2 \times S_1)) = c_1(S_1 \times S_1) \cup c_1(S_2 \times S_2) = (c_0(S_1) \times c_0(S_1)) \cup c_0(S_2) \times c_0(S_1) = S_2' \times S_2' \cup S_1' \times S_2' = X_4'$.

We must extend twice more. For $i = 1, 2, \ldots, 6$ define $c_3(P_i) = P' \cap c_2(X_i)$ and $c_3(C_i) = C' \cap c_3(X_i)$. Thus c_3 maps the set $\{P_i | i = 1, 2, \ldots, 6\} \cup \{C_i | i = 1, 2, \ldots, 6\} \cup \{C,P\}$ to the set $\{P_i' | i = 1, 2, \ldots, 6\} \cup \{C_i' | i = 1, 2, \ldots, 6\} \cup \{C',P'\}$. For example, $c_3(P_3) = P' \cap c_2(X_3) = P' \cap X_4' = P_4'$. It follows that $c_3(P) = P'$, $c_3(C) = C'$, $c_3(P_1) = P_1'$, $c_3(P_2) = P_2'$, $c_3(P_3) = P_4'$, $c_3(P_4) = P_3'$, $c_3(P_5) = P_6'$, $c_3(P_6) = P_5'$, $c_3(C_1) = C_1'$, etc.

Finally, for any expression of the form

$$(*) \qquad (C_{i_1} C_{i_2} \ldots C_{i_{m_k}} P_{j_1} P_{j_2} \ldots P_{j_{n_k}}) \cap (S_c \times S)$$

where each i_a and j_b are members of the set $\{0,3,4\}$, $c \in \{0,1,2\}$, $P_0 = P$, $C_0 = C$ and $S_0 = S$, we define

$$c_4((C_{i_1} C_{i_2} \ldots C_{i_{m_k}} P_{j_1} P_{j_2} \ldots P_{j_{n_k}}) \cap (S_c \times S))$$
$$= (c_3(C_{i_1}) c_3(C_{i_2} \ldots c_3(C_{i_{m_k}}) c_3(P_{j_1}) c_3(P_{j_2}) \ldots c_3(P_{j_{n_k}})) \cap (S_{\bar{c}} \times S)$$

where $\bar{c} = 1$ if $c = 2$, $\bar{c} = 2$ if $c = 1$, and $\bar{c} = 0$ if $c = 0$. Thus C_4 is a mapping from C_o to C_o'. If ρ is a set of relations of the form (*), then by definition $c_4(\rho) = \{c_4(R) | R \in \rho\}$.

We now consider, for ρ, the classes $[R]_\mu$ and $[R]_\phi$ as defined in the preceding section. If $c_4([R]_\mu) \in T_\phi'$ and $c_4([R]_\phi) \in T_\mu'$ for each $[R] \in T$, then c_4 induces a mapping $c: T \to T'$, which is a terminological dual isomorphism. c is called <u>the Copenhagen function</u>.

What the Copenhagen function does in a sense, is to change "the parity" of an expression: for example, the expression $C_3C_4PP_4$ is transformed by c into the expression $C_4C_3PP_3$. Each time that we trimmed the expression CCPP in order to restrict a part of it in terms of male or female, c restricts the expression in the image to the alternative

TABLE III

	T_μ: Crow-type I Male Ego	$c(T_\mu)$: the image of T_μ under c	T'_ϕ: Omaha-type I Female Ego
1.	C_4 C_4CP_4	C_3 C_3CP_3	C_3 C_3CP_3
2.	CP_4 $C_4CP_4P_4$ $C_3CP_3P_4$ $C_3CP_4P_3P_4$ $C_3CP_4P_4P_3P_4$	CP_3 $C_3CP_3P_3$ $C_4CP_4P_3$ $C_4CP_3P_4P_3$ $C_4CP_3P_3P_4P_3$	CP_3 $C_3CP_3P_3$ $C_4CP_4P_3$ $C_4CP_3P_4P_3$ $C_4C_3CP_4P_3$
3.	CP_3 $C_4CP_4P_3$ $C_3CP_3P_3$ $C_3CP_4P_3P_3$ $C_3CP_4P_4P_3P_3$	CP_4 $C_3CP_3P_4$ $C_4CP_4P_4$ $C_4CP_3P_4P_4$ $C_4CP_3P_3P_4P_4$	CP_4 $C_3CP_3P_4$ $C_4CP_4P_4$ $C_4CP_3P_4P_4$ $C_4CP_3P_3P_4P_4$ $C_4C_3CP_4P_3$
4.	P_3 CP_3P_3 $C_4CP_3P_3$ $C_4CP_4P_3P_3$ $C_3CP_3P_3P_3$ $C_3CP_4P_3P_3P_3$	P_4 CP_4P_4 $C_3CP_4P_4$ $C_3CP_3P_4P_4$ $C_4CP_4P_4P_4$ $C_4CP_3P_4P_4P_4$	P_4 CP_4P_4 $C_3CP_4P_4$ $C_3CP_3P_4P_4$ $C_4CP_4P_4P_4$ $C_4CP_3P_4P_4P_4$ $C_3C_3CP_4P_4$
5.	CP_4P_4 $C_4CP_4P_4P_4$ $C_3CP_3P_4P_4$ $C_3CP_4P_3P_4P_4$	CP_3P_3 $C_3CP_3P_3P_3$ $C_4CP_4P_3P_3$ $C_4CP_3P_4P_3P_3$	CP_3P_3 $C_3CP_3P_3P_3$ $C_4CP_4P_3P_3$ $C_4CP_3P_4P_3P_3$
6.	C_3CP_4 $C_3CP_4P_4$ $C_3CP_4P_4P_4$ $C_3CP_4P_4P_4P_4$	C_4CP_3 $C_4CP_3P_3$ $C_4CP_3P_3P_3$ $C_4CP_3P_3P_3P_3$	C_4CP_3 $C_4CP_3P_3$ $C_4CP_3P_3P_3$ $C_4CP_3P_3P_3P_3$ $C_4C_4CP_4P_3$ $C_4C_3CP_3P_3$

choice; for example CCPP $\rightarrow$ C_4CPP restricts which of the two parents of any ego we wish to focus upon. Further restriction yields $C_4CPP \rightarrow C_4C_3PP \rightarrow C_4C_3PP_3$. Now $c(C_4C_3PP_3) = C_3'C_4'P'P_4'$ simply indicates that for each restriction made (in S) on CCPP to obtain $C_4C_3PP_3$ we choose the complementary restriction in S, thereby obtaining $C_3'C_4'P'P_4'$.

To demonstrate the use of the Copenhagen function, we have selected six classes from a Crow-type I system with ego being male and map them into classes from an Omaha-type I system with ego being female. Table III contains the expression for consanguineal relationships with the first column representing a Crow-type I system, containing six classes of relationships taken from Lounsbury's Table 2 (1964:367). Each class is determined by grouping together a kintype in the table which is a focal type with those kin types that can be reduced to that focal type. We then write the relationship between ego and each alter in the notation developed above and put the relationship in a box in the column. For example, we see that M and MZ constitute a class and we write C_4 and C_4CP_4. We assume that MZ can be either MFD or MMD and thus write $C_4CP_4(C_4CP_4 = C_4C_3P_4 \cup C_4C_4P_4)$. The second column contains the result of applying the Copenhagen function to the expressions in the first column. The third column contains the expressions for the relationships between a female ego and alter in an Omaha-type I system taken from Tax's Figure 2 (1955:250).

By comparing the expressions of column 2 with those of column 3 one can see that the Copenhagen function does map each expression from a Crow-type I systems into a corresponding class from an Omaha-type I system. The final expressions in the second, third, fourth, and sixth row of column three have no equivalent expression in column one and two because the table for Crow-type I does not include as comprehensive a list of kin types as the table for Omaha-type I. Were one to use the Copenhagen function on the Omaha-type I expressions, one could expect a Crow-type I system to include the expressions produced by the image of T_ϕ' under c in the appropriate classes.

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