RESEARCH STATEMENT

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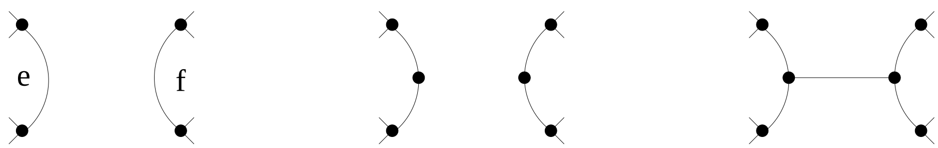
Consider the graph T of the tetrahedron, that is, 4 vertices every two of which are joined by an edge. T is 3-regular since every vertex meets exactly 3 edges; it is 3-connected because it is connected and cannot be disconnected by removing fewer than 3 vertices; and it is planar because it can be drawn in the plane without crossings. Steinitz and Rademacher [6] proved in 1934 that every 3-regular 3-connected planar graph can be constructed from T by repeatedly applying the operation of adding handles illustrated in Figure 1.



(i) e and f are any two edges of G.

(ii) Add a new vertex in each of e and f.

(iii) Add a new edge joining these new vertices.

Figure 1: Adding handles

This theorem is an example of a *generating theorem* because it describes how to construct every graph in some family $\mathcal{F}$ from some subset $\mathcal{F}'$ of the family by some collection $\mathcal{O}$ of operations where, whenever an operation in $\mathcal{O}$ is applied to a graph in $\mathcal{F}$, it produces another member of $\mathcal{F}$. The purpose of this research is to obtain other generating theorems. Ideally, the collection $\mathcal{F}'$ is a small subset of $\mathcal{F}$ and $\mathcal{O}$ consists of a small set of operations.

Consider the two graphs H_1 and H_2 illustrated in Figure 2. The girth of H_1 is 4 since the minimum length of all cycles is 4. The girth of H_2 is also 4. H_1 and H_2 are 3-regular 3-connected planar graphs, so both can be obtained from T by adding handles. Moreover, H_2 contains H_1 *topologically*, that is, H_1 can be obtained from H_2 by deleting some edges or vertices and then replacing some paths by edges. Notice that generating theorems cannot tell us whether there is a construction process starting T going to H_1 and finishing at H_2. *Splitter theorems* can answer such questions.

Let $\mathcal{F}_4$ be the family of 3-connected 3-regular graphs with girth at least 4. Consider the

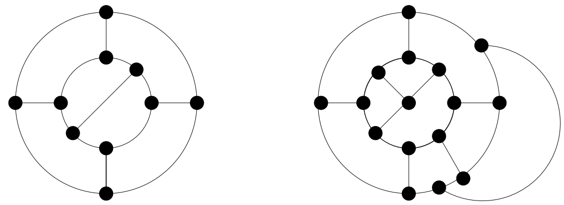


Figure 2: H_1 and H_2(right)

operations of (i) adding handles; (ii) replacing a vertex by $K_{3,3} - v$ (see Figure 3(left)); and (iii) replacing a vertex by $Q - v$ (see Figure 3(right)). The following is a splitter theorem for $\mathcal{F}_4$, which also yields a generating theorem.

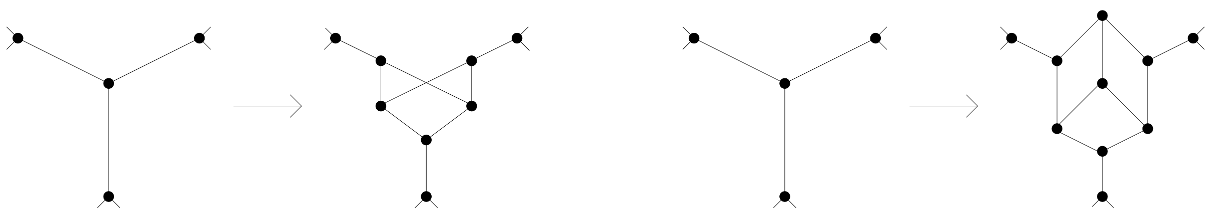


Figure 3: Replacing a vertex by $K_{3,3} - v$ and by $Q - v$

Theorem. *If G and H are 3-connected 3-regular graphs with girth at least 4, and G contains H topologically, then G can be constructed from H within $\mathcal{F}_4$ by adding handles, and by replacing vertices by $K_{3,3} - v$ or by $Q - v$.*

Corollary. *Every 3-connected 3-regular graph with girth at least 4 can be generated from $K_{3,3}$ or Q by adding handles, and replacing vertices by $K_{3,3} - v$ or by $Q - v$.*

The first splitter theorems were derived by Negami [3] for graphs and Seymour [5] for matroids. Results of the same type can also be found in [1], [2], and [4]. In my research, I have obtained, in addition to the last result, splitter theorems for several families of 3-regular and 4-regular graphs. For example, consider the two graphs H_3 and H_4 illustrated in Figure 4. H_3 and H_4 are 4-regular since every vertex meets exactly 4 edges; they are 4-edge connected because each of them is connected and cannot be disconnected by removing fewer than 4 edges. Also, H_3 is *immersed* in H_4 because the vertices of H_3 can be identified with distinct vertices of H_4, and the edges of H_3 can be identified with edge-disjoint paths of H_4.

Let $\mathcal{F}^4$ be the family of 4-edge connected 4-regular graphs. Consider the operations of (i) pinching edges (see Figure 5); and (ii) replacing a vertex by T (see Figure 6). Let K_5 be the graph of 5 vertices every two of which are joined by an edge. K_5 is also a

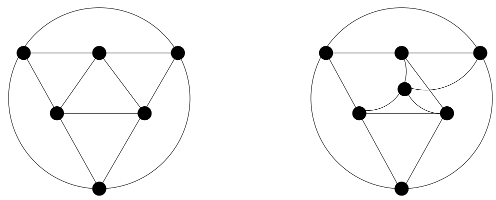


Figure 4: H_3 and H_4(right)

4-edge connected 4-regular graph. The following are a splitter theorem and a generating theorem for $\mathcal{F}^4$.

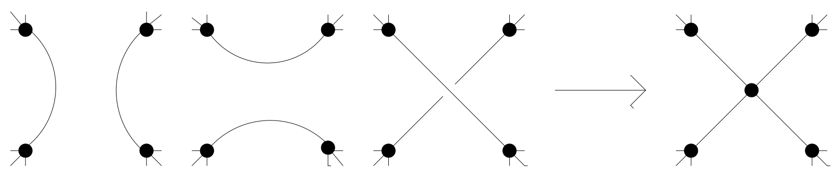


Figure 5: Pinching edges

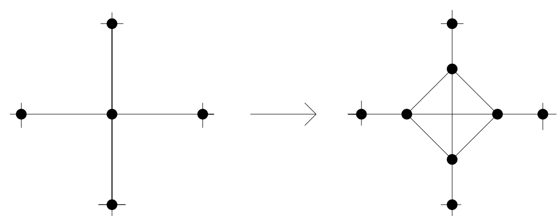


Figure 6: Replacing a vertex by T

Theorem. *If G and H are 4-edge connected 4-regular graphs, and H is immersed in G, then G can be constructed from H within $\mathcal{F}^4$ by pinching edges and replacing vertices by T.*

Corollary. *Every 4-edge connected 4-regular graph can be generated from K_5 by pinching edges and replacing vertices with T.*

Both generating theorems and splitter theorems provide useful structural information for classes of graphs and the aim of my contunuing research is to derive such results for several more families of graphs.

References

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