Lab 3: Introduction to Complex Impedance

Goal

The goal is to understand complex impedance and generalized voltage dividers using reactive components.

Objectives

1. Observe the frequency dependence of voltage divider output using a RC circuit

2. Observe the phase shift between input and output signal due to complex impedance.

3. Compare oscilloscope measurements of voltage divider output and phase shift with calculated values.

4. Simple applications of RC circuits

Expectations

1. You are expected to take detailed notes during each step outlined in the procedure that can be used during the lab report write-up.

2. You are expected to provide a neat table of the data that you measured where you clearly label what each data set is and include units for all measured quantities.

3. You are expected to clearly record the measured values of any components that you use.

4. You are expected to clearly record the detail related to images captured by the oscilloscope.

5. You are expected to make your final plots in a program such as Excel. Make sure that your data points appear clearly on the plots, that all axes are clearly labeled and have units.

6. If it is possible to compare your measurements with an expectation or a prediction, you are expected to do so in your lab report

7. You are expected to answer the questions encountered in this manual as well as discuss exercises given during the lectures in your lab write up.

Introduction to Concepts

An unloaded high-pass filter (RC circuit) is shown in Figure 1-Left. The impedance of the high-pass filter is shown using a phasor diagram in Figure 1-Center and the frequency response of the circuit is shown in Figure 1-Right. The output of the circuit is approximately equal to the input at high frequencies and goes to zero at low frequencies. The frequency at which signal attenuation is −3 dB is referred as −3 dB "breakpoint". It's the frequency at which $V_{out}/V_{in} = 1/\sqrt{2} \sim 0.707$,

$$f_{3dB} = \frac{1}{2\pi RC}$$

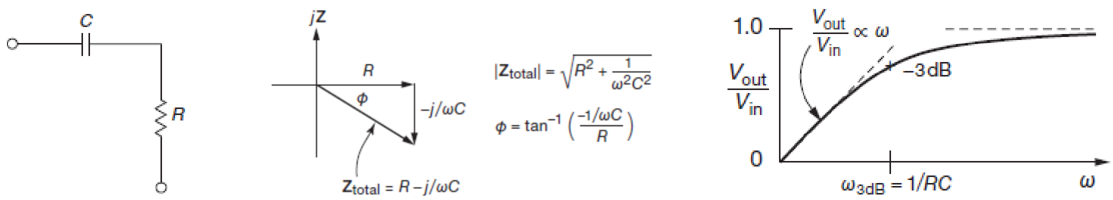

Figure 1: Left: Unloaded high-pass filter Center: Phasor diagram and phase shift of the RC circuit. Right: The frequency response of the RC circuit and it confirm the circuit is a high pass filter [1]

Experimentally, the phase shift of the RC circuit can be measured using an oscilloscope. For our lab, we will construct a low-pass filter RC circuit as shown in Figure 2. The shift in time scale is measured using the oscilloscope as shown in the Figure 3. The phase shift, ϕ,

$$\begin{aligned}\phi &= \frac{\Delta t}{T} \cdot 360^o \\ &= f\Delta t \cdot 360^o \end{aligned} \tag{1}$$

Where T is the period of the signal (V_{in} or V_{out}) and $T = 1/f$, f is the frequency and the Δt is the shift between V_{in} and V_{out}

A simple voltage divider is constructed using capacitor and resistor as shown in Figure 2. The complex Ohm's law gives,

$$\boldsymbol{V}_{out} = \frac{V_{in}[R + j/\omega C]R}{R^2 + (1/\omega^2 C^2}$$

Where R is the resistance, C is the capacitance, $\omega = 2\pi f$, and $\boldsymbol{V}_{out}$ is a complex number. The amplitude of the output signal is the magnitude of the complex $\boldsymbol{V}_{out}$,

$$\begin{aligned} V_{out} &= (\boldsymbol{V}_{out}\boldsymbol{V}^*_{out})^{1/2} \\ &= \frac{R}{[R^2 + (1/\omega^2 C^2)]^{1/2}} \cdot V_{in} \\ &= \frac{2\pi fRC}{[1 + (2\pi fRC)^2]^{1/2}} \cdot V_{in} \end{aligned}$$

Preliminary Lab Questions

You will find useful to complete the preliminary lab questions before starting the procedure.

1. Calculate the V_{out}/V_{in} at f_{3dB} for the circuit in Figure 2

2. Calculate the phase shift at f_{3dB}

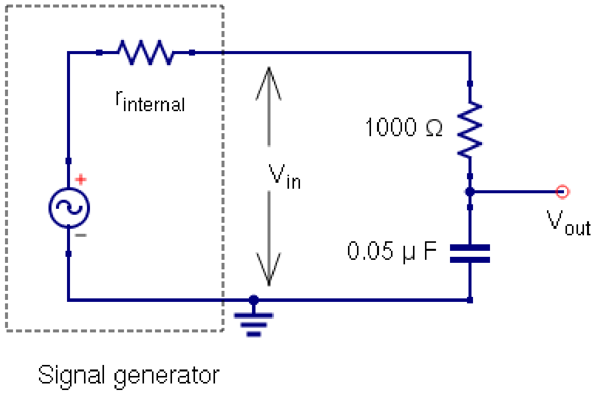

Figure 2: This RC circuit acts as a low pass filter. It can be considered a generalized voltage divider using a capacitor and a resistor connected in series.

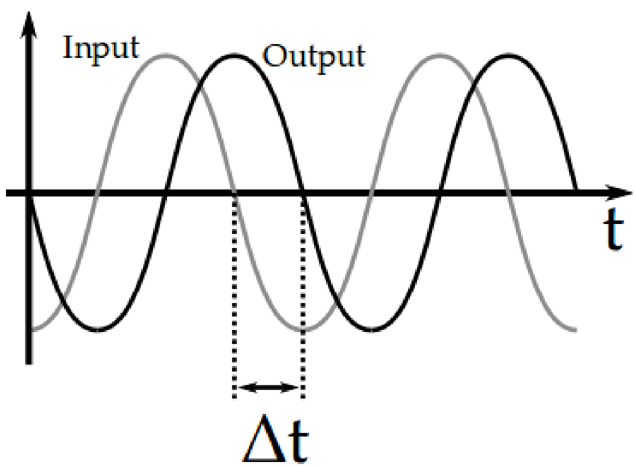

Figure 3: The phase shift in time axis (Δt) is measured as shown.

Equipment and Parts

1. Signal generator (Instek GFG-8216G)
2. Oscilloscope (RIGOL DS11002E)
3. Multimeter
4. RLC meter
5. RC box

You will also need the following components in order to carry out this lab.

1. 0.01 μF capacitor
2. 10 KΩ resistor
3. 110 V to 6.0 V transformer
4. Few pairs of banana cables
5. Two BNC to crocodile cables
6. BNC T-junction to banana cable

Procedure

- Set the signal generator to sine-wave and set the frequency to f_{3dB} for the circuit in Figure 2
- Using the oscilloscope set the peak amplitude to 2.8V
- Measure resistance and capacitance using the RCL meter and use these accurate values instead of face values shown in the RC box
- Set the resistance and capacitance values shown in Figure 2 using the RC box
- Build the circuit shown on Figure 2
- Connect the input and output of the circuit to the channel 1 and 2 of the oscilloscope, respectively
- Capture oscilloscope image with input and output signals
- Configure the oscilloscope to generate trace similar to Figure 4
- Measure the peak amplitudes for V_{in} and V_{out} signals. These are the magnitudes of V_{in} and V_{out} signals that can be calculated using the equations discussed in the Section "Introduction to Concepts"
- Before proceeding to next steps compare V_{out}/V_{in} value with expected value at f_{3dB}

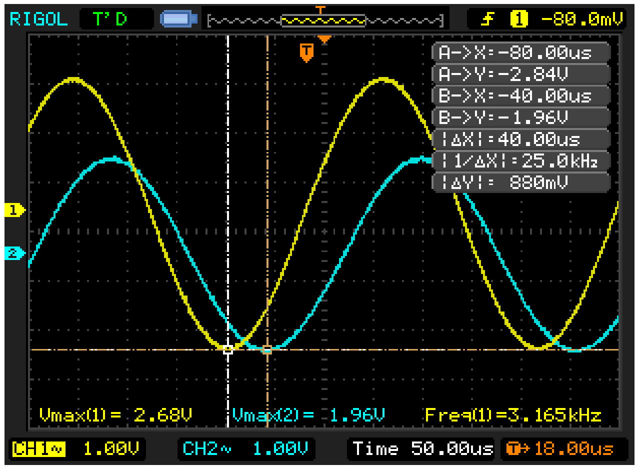

Figure 4: Connect the V_{in} and V_{out} to the scope. The goal is to measure amplitudes and phase shift.

- Results should agree within 10%
- Measure the Δt between V_{in} and V_{out} signals
- It is recommended to set the vertical scale of the two channels to 0.5V/unit for both channel 1 and 2 before measuring the Δt (see Figure 5)
- Compute the phase shift between V_{in} and V_{out} signals at f_{3dB}
- Before proceeding to next steps compare measurement and calculation of phase shift
- Results should agree within 10%
- If your V_{out}/V_{in} and phase shift results disagree beyond the 10%, repeat the measurements.
- Vary frequency and measure V_{in}, V_{out}, and Δt for frequencies shown in the sample Table 1 using the oscilloscope.
- Deduce the f_{3dB} from the data table
- Discuss any discrepancy observed in deduced f_{3dB} vs. actual f_{3dB}
- Plot V_{out}/V_{in} vs. frequency
- Determine f_{3dB} from the plot
- Plot ϕ vs. frequency
- Plot theoretical values of V_{out}/V_{in} vs. frequency and ϕ vs. frequency
- Overlay measured and theoretical plots

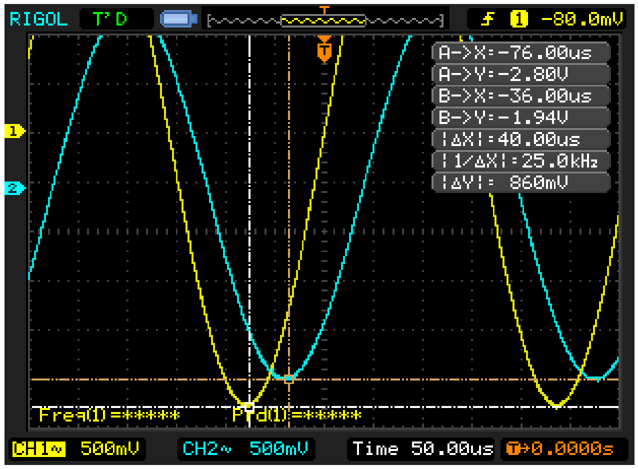

Figure 5: Set the vertical scales of the two channels to 0.5V/unit for both channel 1 and 2 before measuring the Δt

Table 1: Frequency dependent attenuation and phase shift

f (Hz)	V_{in} (V)	V_{out} (V)	Δt (μs)	V_{out}/V_{in}	ϕ (deg)
100					
200					
400					
800					
1600					
3183					
3200					
6400					
9600					
12800					
19200					

RC Circuit Application: Selecting Signal from Signal Plus Noise

A high-pass filter is used to separate a high frequency signal from the power line noise as shown in the Figure 6. The transformer adds a large 60 Hz sine wave (peaked at about 6 V) to the output of the sin wave from the signal generator.

1. Build the circuit shown in Figure 6
2. Observe the output from the signal generator (at A in Figure 6)
3. Set the signal generator amplitude to maximum possible value.
4. Measure the amplitude and frequency at A
5. Perform a Fourier analysis of the waveform trace using 'MATH' function of the oscilloscope
6. Capture the Fourier signal
7. Capture the traces showing this signal (show frequency and amplitude in the saved image)
8. Observe the output after the transformer (at B in Figure 6)
9. Measure the amplitude and frequency at B
10. Capture the traces showing this signal (show frequency and amplitude in the saved image)
11. Perform a Fourier analysis of the waveform trace using 'MATH' function of the oscilloscope
12. Capture the Fourier signal
13. Capture the output at C in Figure 6 (show frequency and amplitude in the saved image)
14. Measure the amplitude and frequency at C
15. Perform a Fourier analysis of the waveform trace using 'MATH' function of the oscilloscope
16. Capture the Fourier signal
17. Explain signals seen at A, B and C using the RC filter circuit knowledge

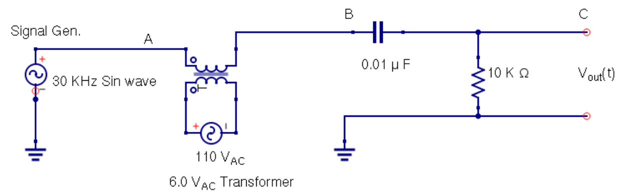

Figure 6: Composite signal, consisting of two sine waves

Additional Lab Questions

You will find useful to complete the preliminary lab questions before starting the procedure.

1. Derive the equation for $\boldsymbol{V}_{\text{out}}$ (complex) for the RC circuit used in the lab
2. Derive the equation for the magnitude of the output signal, V_{out}
3. What is the effect of including or excluding the internal resistance of the signal generator? Does the adding internal resistance reduce the discrepancy of your results?
4. Why the RC circuit shown in Figure 2 is called a "low pass filter"

References

[1] Horowitz and Hill. *Art of Electronics*. Cambridge University Press, 2015.