Lab 4: Resonant Circuits

Goal

The goals include apply resonant circuit to find Fourier components of a square wave and analyze the resonant circuits with LCR resonator.

Objectives

1. Construct LCR resonant circuit, measure resonance frequency and Q factor
2. Utilize the LCR circuit as a "Fourier Analyzer"

Expectations

1. You are expected to take detailed notes during each step outlined in the procedure that can be used during the lab report write-up.
2. You are expected to provide a neat table of the data that you measured where you clearly label what each data set is and include units for all measured quantities.
3. You are expected to clearly record the measured values of any components that you use.
4. You are expected to clearly record the detail related to images captured by the oscilloscope.
5. You are expected to make your final plots in a program such as Excel. Make sure that your data points appear clearly on the plots, that all axes are clearly labeled and have units.
6. If it is possible to compare your measurements with an expectation or a prediction, you are expected to do so in your lab report
7. You are expected to answer the questions encountered in this manual as well as discuss exercises given during the lectures in your lab write up.

Introduction to Concepts

LC Resonant Circuit

The parallel LC circuit shown in Figure 1 is highly selective, passing a narrow range of frequencies. The opposite behaviors of an inductor and a capacitor, the impedance of the parallel LC goes to infinity at the resonant frequency,

$$\mathrm{f_o} = \frac{1}{2\pi\sqrt{\mathrm{LC}}}$$

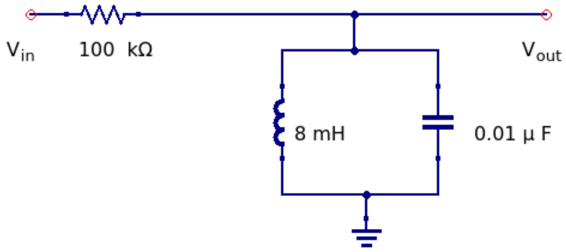

Figure 1: The parallel LC resonant circuit.

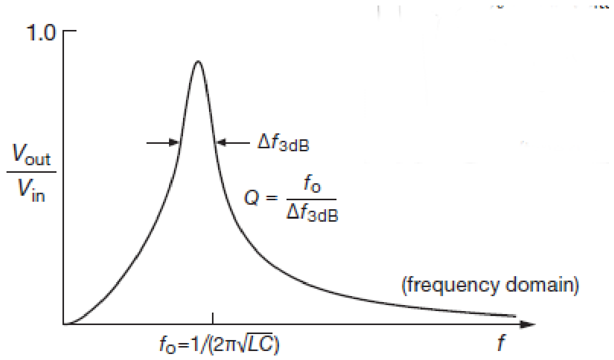

Figure 2: Frequency response of parallel LC circuit.

giving a peak in the response there. The circuit response is as shown in Figure 2. This feature is utilized to confirm one of Fourier's claims where Fourier series for a square wave,

$$\mathrm{SW}(\omega \mathrm{t}) = \frac{\pi}{4}(\sin(\omega \mathrm{t}) + \frac{1}{3}\sin(3\omega \mathrm{t}) + \frac{1}{5}\sin(5\omega \mathrm{t}) + \frac{1}{7}\sin(7\omega \mathrm{t}) \cdots)$$

Where $\mathrm{SW}(\omega \mathrm{t})$ is the square wave amplitude at $\omega = 2\pi \mathrm{f}$ frequency. The amplitudes and frequencies of Fourier components of the square wave is shown in the Figure 3. The resonant circuit (Figure 1) can serve as a "Fourier Analyzer" by measuring the amount of $\omega_{\mathrm{o}} = \mathrm{n} \times \omega$ where n is the integer in the above $\mathrm{SW}(\omega \mathrm{t})$ equation and $\omega_{\mathrm{o}} = 2\pi \mathrm{f}_{\mathrm{o}}$ is the resonant frequency of the circuit. The Fourier components of the square wave at the resonant frequency of the circuit is shown in Figures 4 and 5

Preliminary Lab Questions

You will find useful to complete the preliminary lab questions before starting the procedure.

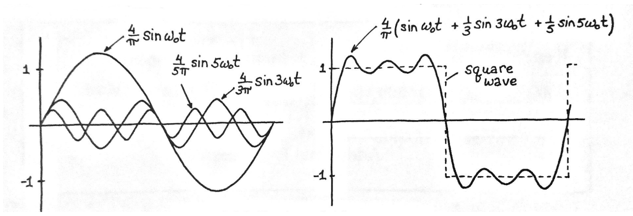

Figure 3: Fourier series for square wave, $\mathrm{SW}(\omega \mathrm{t})$.

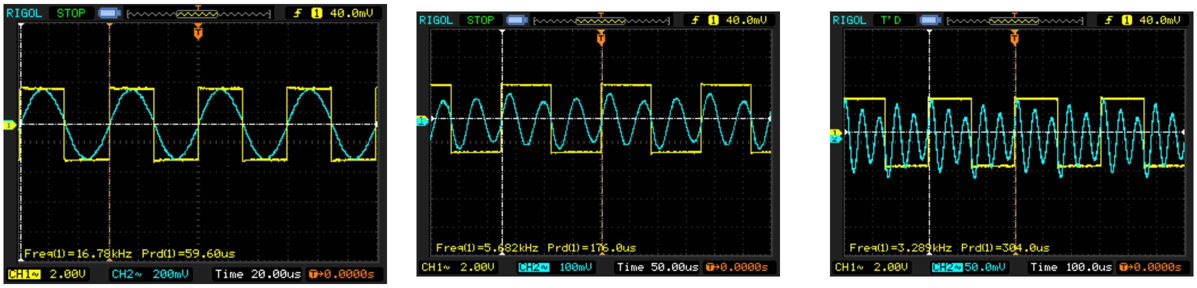

Figure 4: The Fourier components of the square wave observed through the resonant circuit. The $n = 1$ the fundamental, $n = 3, 5$ harmonics are shown. Notice there are n no. of sine-wave peaks within the square wave period and the harmonics are decaying.

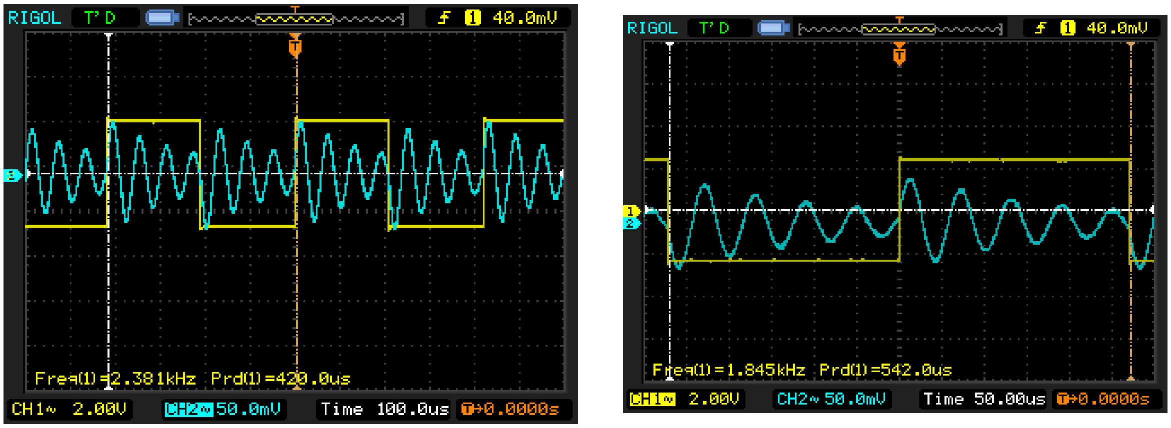

Figure 5: The Fourier components of the square wave observed through the resonant circuit. The $n = 7, 9$ harmonics are shown. Notice there are n no. of sine-wave peaks within the square wave period and the harmonics are decaying.

1. Calculate the resonant frequency of the circuit in Figure 1
2. Calculate the phase offsets at $f_{-3\text{dB}}$ points and at f_o using a Phasor diagram or the equation we discussed in the lecture for the resonant circuit in Figure 1

Equipment and Parts

1. Signal generator (Instek GFG-8216G)
2. Oscilloscope (RIGOL DS11002E)
3. Multimeter
4. RLC meter
5. proto-board

You will also need the following components in order to carry out this lab.

1. 0.01 μF capacitor and 100 KΩ resistor for the resonant circuit.

2. RLC board for the inductor in the resonant circuit.
3. Two pairs of crocodile cables
4. Connecting wires (4 - 6)
5. Three BNC to crocodile cables

Procedure

LCR Frequency Response

- Using RCL meter, measure the capacitance, inductance and resistance of the components for the circuit shown in Figure 1
- Calculate the resonant frequency f_o
- Build the circuit shown in Figure 1
- Connect the signal generator as input
- Connect both input and output signals to the oscilloscope
- Set the signal generator to sine-wave and set the frequency to f_o
- Set the oscilloscope channels to “AC” mode
- Using the oscilloscope set the peak amplitude to 2.8V
- The goal is to reproduce Figure 2 using data, therefore determine the range of frequencies to take data
- Record your data in a table similar to Table 1
- Calculate the Q factor for the resonant circuit.
- Measure the phase shift between input and output signals at f_{-3dB} points
- Measure the phase shift between input and output signals at f_o
- Calculate the phase shifts at f_{-3dB} points and at f_o using a Phasor diagram or the equation we discussed in the lecture
- Estimate percent errors and discuss deviations

Table 1: Frequency dependent attenuation and phase shift

f (Hz)	V_{in} (V)	V_{out} (V)	V_{out}/V_{in}

Fourier Analysis with LCR Resonant Circuit

- Set the signal generator to **square-wave** and set the frequency to f_o
- Set the oscilloscope channels to "AC" mode
- Using the oscilloscope set the peak amplitude to 2.8V
- Build the circuit shown in the Figure 1
- Connect the input and output to the channel 1 and 2 of the oscilloscope, respectively
- Save the input and output from the oscilloscope (see Figure 4)
- Measure the output amplitude of the fundamental sine-wave
- Record fundamental sine-wave frequency and output amplitude in the first raw of the Table 2
- Gradually lower the driving (input) frequency of the square wave until you get another peak response (it should occur at 1/3 of the resonant frequency) and this is the third harmonic of the square wave.
- Record third harmonic frequency in the Table 2
- Record the amplitude of the third harmonic sine-wave in the Table 2. The amplitude of the third harmonic sine-wave should be 1/3 of the amplitude of the fundamental frequency. With some care, you will be able to make an accurate measurement of the amplitude.
- Repeat harmonic frequency and amplitude measurements for next three harmonic frequencies and record in the Table 2.
- Calculate frequency and amplitude ratio for your measurements
- Results should agree with expected frequency and amplitude ratio based on Fourier series as shown in the Table 3.
- Calculate the uncertainty (percent difference) of your measurements.

Table 2: Fourier Series Sine-wave Components of a Square Wave

Harmonic (n)	f_{harmonic} (kHz)	Amplitude (mV)	$\frac{f_{\text{fundamental}}}{f_{\text{harmonic}}}$	$\frac{\text{Amplitude}_{\text{harmonic}}}{\text{Amplitude}_{\text{fundamental}}}$
1 (fundamental)				
3				
5				
7				
9				

Table 3: Frequency and Amplitude Ratio for Fourier Series Sine-wave Components of a Square Wave

Harmonic (n)	$\frac{f_{\text{fundamental}}}{f_{\text{harmonic}}}$	$\frac{\text{Amplitude}_{\text{harmonic}}}{\text{Amplitude}_{\text{fundamental}}}$
1 (fundamental)	1	1
3	3	$\frac{1}{3}$
5	5	$\frac{1}{5}$
7	7	$\frac{1}{7}$
9	9	$\frac{1}{9}$

Additional Lab Questions

You will find useful to complete the lab questions before starting the lab.

1. Why the observed sine-waves through the resonant circuit decay?
2. Explain why a resonant LC circuit can be used as a “Fourier Analyser”?

References