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Reduction of Coherent Scattering Noise with Multiple Receiver Doppler

Steven A. Jones and Kiran Krishnamurthy
Louisiana Tech University
Biomedical Engineering
Ruston, LA, USA

Corresponding Author:
Steven A. Jones
Louisianana Tech University
PO Box 10348
Ruston, LA 71272, USA
Phone: (318) 257-2288
Fax: (318) 257-2562
Email: sajones@coes.latech.edu

Running Title: Enhanced Resolution Doppler by Multiple Receivers

Mr. Krishnamurthy is currently at Kodak Health Imaging

Abstract
Doppler ultrasound velocity estimates are inherently subject to error as a result of both Doppler ambiguity and coherent scattering. The coherent scattering error is a result of changes in the phase of the returned echo as particles enter and leave the sample volume. This phase depends on the distance from the transmitter to the scatterer and then to the receiver. This distance, in turn, depends on the angle of the receiver. A numerical simulation has been used to determine whether velocity estimates obtained from receiver probes at different angles are independent of one another. If so, then it is possible to obtain an improved velocity estimate from the combination of several receivers at different angles. The simulation results show that the cross-correlation between velocity estimates is reduced to 0.3 when receiver probes are oriented 5° apart. These results suggest a new Doppler method that can significantly reduce velocity estimation error.

Introduction
Whereas an ideal Doppler device would provide an exact velocity measurement at an infinitely small location in space, practical measurements must be obtained from a finite region in space, and are subject to an inherent estimation error. Fundamental ambiguity principles indicate that, for a given Doppler method, a decrease in the sample volume size will reduce velocity accuracy (Woodward, 1953). A fundamental increase in the accuracy of Doppler measurements can thus come only if additional information about the velocity field can be obtained. An example is wide band processing, in which the data contained in the envelope of the received signal, or equivalently in the signal harmonics, are used to improve Doppler estimation. Another example that needs to be investigated is the use of multiple receivers. Because the target particles scatter sound in all directions, it can be postulated that sound received from different receiver

angles will have complementary information that can be used to improve estimation. This postulate will be investigated in this paper through the use of a simulation method.

An enhanced-resolution Doppler method would have multiple benefits in cardiovascular diagnosis. Depending on the needs of the application, the improvement could be applied to increased spatial resolution, increased temporal resolution, increased signal to noise ratio, or any combination of these three. For example, the most straightforward method for estimating the regurgitant volume in valvular insufficiency is integration of the blood velocity over the orifice area. In practice, this method is difficult because orifice diameters are often small in comparison to the Doppler sample volume (Mizushige et al., 2000). Enhanced spatial resolution would make this method more accurate. Also, the classification of carotid artery stenosis severity by duplex ultrasound is still inaccurate (Grant et al., 2000; Perkins et al., 2000). The ability to measure high frequency fluctuations in the post-stenotic flow field can provide valuable information because the frequencies of these fluctuations are related to the stenosis diameter and the flow rate (Gupta et al, 1975; Jones and Fronek, 1987). Typical values of these frequencies range from 100 to 1000 Hz (Jones and Fronek, 1988), which cannot be accurately captured with conventional Doppler methods. Improved resolution would help in diagnosis of coarctations, malformations, and other vascular anomalies.

The limitations on Doppler ultrasound are caused by Doppler ambiguity and coherent scattering (Jones, 1993). Doppler ambiguity includes spectral broadening and spatial ambiguity. Spectral broadening means that the returned spectrum will have power at frequencies other than the Doppler shift, and spatial ambiguity means that the location of the measurement is not a single point, but a distribution in space. Without spectral broadening, the spectrum of the Doppler signal would be a single infinitely narrow peak at the Doppler frequency, which would unambiguously indicate a single velocity. Some broadening is caused by flow effects, such as shear and turbulence (Garbini et al., 1982a, 1982b), but a significant component is caused by the interaction of the scattering particles with the beam pattern and gating (Newhouse, 1988). This component is present even when velocity is uniform throughout the sample volume. Although spectral broadening is troublesome, coherent scattering amplifies its effect on the inability to accurately estimate velocity. Coherent scattering transforms a smooth, broadened spectrum into a ragged spectrum, and it is caused by the random phases between signals from different scatterers within the sample volume (Jones , 1993). This effect is illustrated in Fig. 1.

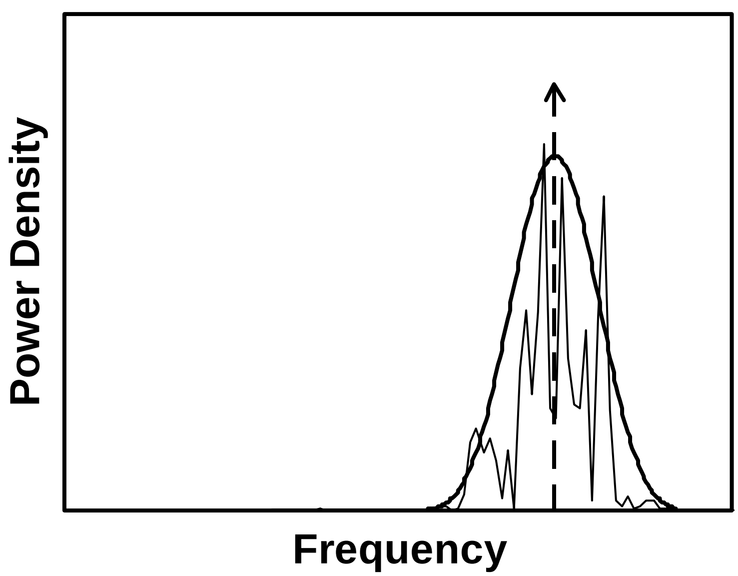


Figure 1: Illustration of spectral broadening and coherent scattering. The dashed arrow is the ideal spectrum. The broadened spectrum (thick line) is a smooth curve. Coherent scattering adds a randomness to the spectrum (thin line).

There have been numerous attempts to circumvent the inherent limitations of Doppler ultrasound. One method is to change the nature of the transmitted signal, as in random (Bendick and Newhouse, 1974) and pseudorandom (Cathignol et al., 1980) Doppler. While these techniques have some advantages over conventional methods, ultimately they do not reduce Doppler ambiguity, but rather redistribute it in frequency and space. Modern spectral analysis methods can also help to reduce the variance in Doppler estimations (David et al., 1991; Fort et al., 1995; Wang and Fish, 1996), but only when the underlying spectral model is appropriate to the velocity measurement (Kay and Marple, 1981).

Methods that do show fundamental improvement in the Doppler technique are the wide band methods adapted for ultrasound in the late 80's and early 90's (Bonnefous and Pesqué, 1986; Embree and O'Brian, 1991; Ferrara and Alghazi, 1991a; Ferrara, 1991a and 1991b, Foster et al., 1991). Narrow band methods use only the fundamental of the downmixed returned echo. Wide band methods take advantage of additional information in the higher harmonics, which relate to the shape of the envelope of the returned echoes. Wide band methods provide an improved velocity estimate because they use more information from the velocity field than does conventional Doppler.

The use of multiple receivers for Doppler ultrasound has been proposed for several purposes. Investigators have used three-probe systems to deduce the complete blood velocity vector. This requires measurements from three non-coplanar probes (Dotti et al., 1992; Overbeck et al., 1990; Tamura et al., 1990). The intention of these studies is not to decrease the coherent scattering, however. More recently, Yasser and Twefik (1995) have described a technique that uses multiple transceivers. Their technique alters the ambiguity function of the system, but the addition of each transceiver generates ambiguities in locations of the distance-frequency plane that were previously unambiguous.

Three-dimensional Doppler combines multiple color Doppler image planes to form a three-dimensional reconstruction of the entire flow field. The purpose of this method is to obtain a complete representation of the velocities within the artery volume. There is no attempt to improve on the resolution over conventional Doppler systems.

Because coherent scattering is a stochastic process (Censor and Newhouse, 1988), some improvement in the measurement can be expected if multiple measurements are taken and averaged together. The randomness in the signal is caused by the differing phases of the signals from individual scattering particles, and these phases depend on the differences in transmitter-to-receiver pathlength for all of the scattering particles. These pathlength differences depend, in turn, on the angle from which the measurement is taken. Consequently, two receivers placed at different angles from the transmitter will be subject to different random fluctuations in the received signals. In the best case, these signals would become completely uncorrelated for a relatively small angle, thus allowing multiple observation of the same sample volume such that the randomness could be averaged out of the measurement. Whereas it is clear that this must work to some extent, the usefulness of this idea depends on the degree to which the signals become uncorrelated for a given angle difference. In an extreme case, if the stochastic noise from two receivers placed 1° from each other were totally uncorrelated, it would be possible to reduce noise in the velocity estimate by a factor of 10 with a 10 by 10 array that has a single transmitter and 100 receivers evenly spaced from one another over a 10° arc.

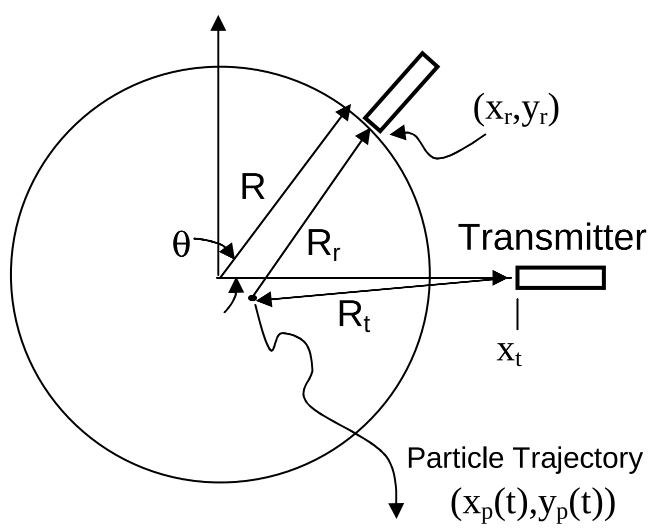
To determine the relationship between correlation and receiver angle, the returned signals from different probes placed at various angles from the transmitting probe have been simulated.

Velocity estimates are computed as a function of time for each receiver, and the cross-correlations between the resulting time signals are calculated for each separate angle.

Methods
The simulation model is based on the procedure used by Azimi and Kak (1985) and by Bonnefous and Pesqué (1986). The signal from the receiving probe is the sum of signals from individual particles. The geometry that describes the simulation is shown in Fig. 2.
The transmitter is oriented along the horizontal axis at a distance x_t from the origin. The receiving sensor is at (x_r, y_r) and oriented at an angle θ from the axis of the transmitting probe. The particle is at a position ($x_p(t)$,$y_p(t)$), which changes with time. Sound is sent from the transmitter to the particle along the path R_t, and returns to the probe along path R_r. The time delay of the received signal depends on the path length $L_p = R_t + R_r$. In terms of the particle position, $R_t = \sqrt{(x_t - x_p(t))^2 + y_p(t)^2}$, and $R_r = \sqrt{(x_r - x_p(t))^2 + (y_r - y_p(t))^2}$. In terms of the angle θ of the receiver from the horizontal axis, $x_r = R\cos\theta$, and $y_r = R\sin\theta$. Hence:

$$L_p = R_t + R_r = \sqrt{(x_t - x_p(t))^2 + y_p(t)^2} + \sqrt{(R\cos\theta - x_p(t))^2 + (R\sin\theta - y_p(t))^2} \quad \textbf{eqn (1)}$$

Figure 2: Sound scattering geometry for the simulated signal. The curved trajectory is the path that a particle takes as $x_p(t)$ and $y_p(t)$ change with time. Changes in particle position change the transmit-receive distance and hence the phase of the detected signal.



If the transmitted signal has the form $A_t(t)\cos(\omega t)$, where $A_t(t)$ is a time-varying amplitude, then the signal at the receiver is $A_t(t)S(\theta)V(x_i, y_i)\cos(\omega t - kL_p)$, assuming a far field approximation where phase is proportional to path length. The parameter $k = 2\pi/\lambda$ is the wave number for the transmitted frequency. The angle factor $S(\theta)$ is included to recognize that the amplitude of scattering from a particle depends on the angle at which it is measured. The particle location factor $V(x_i, y_i)$ is needed because the amount of incident energy on the particle depends on the particle's location within the sample volume. The sample volume depends on the beam patterns of both transducers and on the gating. It is the particle location factor that causes the ambiguity effects. Attenuation due to radial spreading and sound absorption is neglected, but could readily be absorbed in the amplitude factor. The total signal at the receiver is the summation of the signals from all scatterers, hence:

$$s_r(t) = \sum_p A_t(t) S(\theta) V(x_i, y_i) \cos(\omega t - kL_p).$$ **eqn (2)**

With this model, a series of time domain quadrature Doppler signals has been simulated for various angles of the receiver probe. The power spectra of 8 ms segments (256 data points per record) of the resulting data were computed by complex fast Fourier transform. The peak of each spectrum was found by a global search of the absolute maximum. This yielded a series of velocity estimates $\hat{v}_i(t_i, \theta)$ for each segment. The same set of random particles was used for all values of θ. The cross correlation between velocity signals from different angles was computed from:

$$C(\varphi) = \frac{\sum_{i=1}^{n} [\hat{v}_i(t_i,\theta) - v(\theta)][\hat{v}_i(t_i,\theta+\varphi) - v(\theta+\varphi)]}{\sqrt{\sum_{i=1}^{n} [\hat{v}_i(t_i,\theta) - v(\theta)]^2 \sum_{i=1}^{n} [\hat{v}_i(t_i,\theta+\varphi) - v(\theta+\varphi)]^2}}$$ **eqn (3)**

The values $v(\theta)$ are the values of velocity that were input into the simulation, corrected for the receiver probe angle, θ. Since, for the configuration used in the simulation, the Doppler shift obtained from a probe at angle θ is $f_d = v f_0 (1 + \cos\theta)/c$, the estimated velocity at angle θ must be divided by $(1 + \cos\theta)/2$ so that it can be compared to the velocity estimate at $\theta = 0$. Since the present simulation uses a constant velocity, $v(\theta)$ is independent of time. Equation 3 yields a value of 1 if $\hat{v}(t_i, \theta)$ and $\hat{v}(t_i, \theta + \varphi)$ are identical and a value of 0 if they are completely independent. A value of –1 is obtained if $\hat{v}(t_i, \theta) = -\hat{v}(t_i, \theta + \varphi)$.

Table 1 shows the parameters used in the simulations to be presented below. The positions of the particles are random, evenly distributed numbers. As a first approximation, the beam patterns of the transmit and receiver probes are ignored. It is assumed that all particles within the sample volume return signals to the receiver whose amplitudes depend on the particle's location in the sample volume. A triangle function is used for this weighting. The triangle is the weighting that would be obtained for a standard pulsed Doppler device with identical transmitting and receiving probes, assuming that the range gate duration is identical to the transmitted pulse duration (Jones and Giddens, 1990). This approximation means that the transducer beam patterns are being ignored to ensure that sound received at both transducers comes from

Parameter	**Value**
Pulse Repetition Frequency (f_p)	32000 Hz
Sound Speed (c)	158,000 cm/sec
Time Duration (T)	2.16 sec
Carrier Frequency (f_0)	10 MHz
Particle Velocity (v)	60 cm/sec
Average Number of Particles in Sample Volume (N_p)	15,400
Sample Volume Length	2 mm
Sample Volume Width	1 mm
Distance from Transducer to Sample Volume	2 cm

Table 1: Parameters used in the simulation

the same group of particles. If sound came from a different group of particles, it would not be an adequate test of signal independence. The weighting laterally across the sample volume (along the y-axis) is uniform over a diameter of 1 mm.

Results

Figure 3 shows the in-phase signals from two simulations with different receiver angles. Both signals show the amplitude and phase modulation that is typical of Doppler ultrasound signals. It is notable that although both signals are generated from identical scatterer configurations, they show different amplitude modulation envelopes as a result of the 5° difference in angle. The scale for the "Voltage" axis is arbitrary.

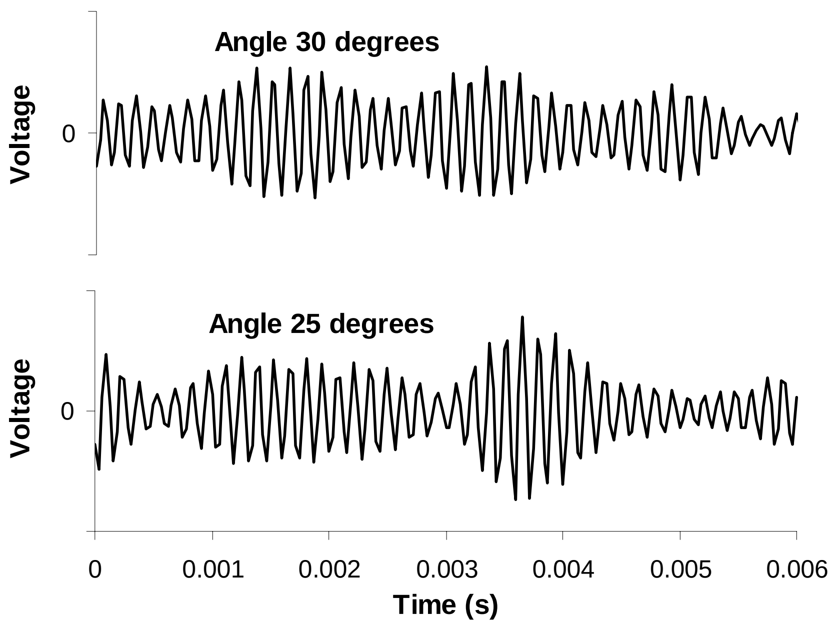


Figure 3: Two in-phase signals from the same configuration of particles but detected by different receivers separated by 5 degrees. Differences can be seen in both the phase and amplitude modulation of the signals, indicating a degree of independence.

When peak tracking is used on the quadrature signals, a trace of velocity as a function of time is obtained. Figure 4 shows estimated velocity as a function of time for scattered sound received by two probes at 25° and 30° from the transmitting probe. The input value for velocity is 60 cm/s. The mean values of the velocity estimates are 59.9 cm/s (25° receiver) and 59.6 cm/s (30° receiver). The random noise in the trace is clearly not identical for the two simulations, indicating some independence of the two signals.

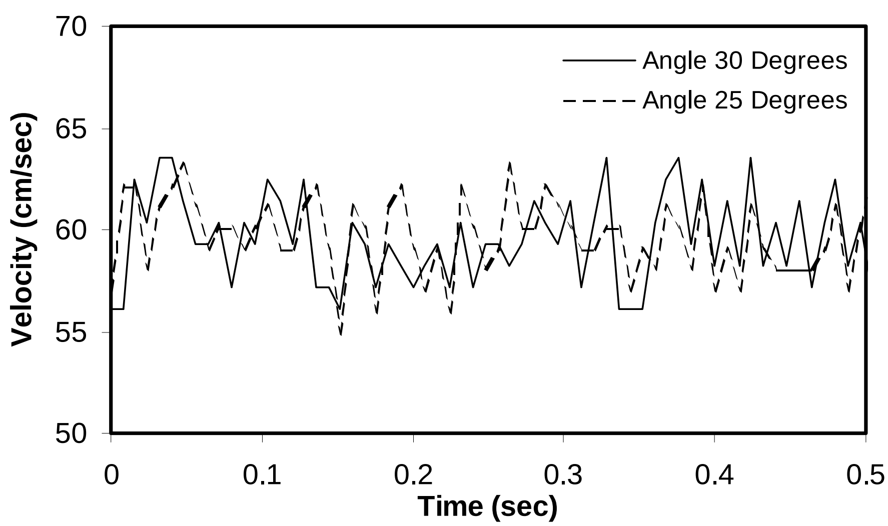


Figure 4: Estimated velocity as a function of time for two receiver probes separated by 5 degrees. The differences in the velocity noise levels indicate that these two traces contain complementary information that can be used to improve velocity estimates.

The cross-correlation between velocity estimate time-sequences is shown as a function of the angle of the receiver probe in Fig. 5. For each receiver angle, the time-sequence is cross-correlated with the signal received at an angle of 30° according to eqn 3. Thus, for an angle of 30° the correlation is 1. The correlation decreases as the receiver angle increases or decreases from 30°. For angle deviations of 5°, the correlation is approximately 0.3.

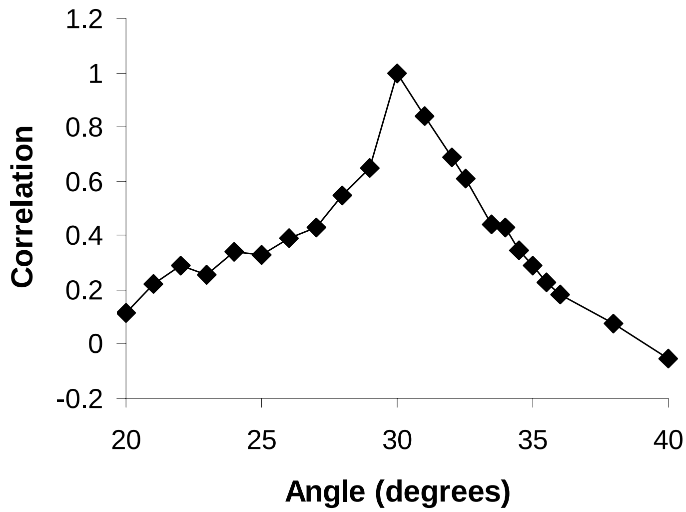


Figure 5: Cross correlation between time traces of the velocity estimate. Signals are simulated for receiver probes oriented at angles ranging from 20 to 40 degrees from the transmitting probe, and velocity estimates are obtained from calculation of the power spectrum followed by peak tracking. Each velocity sequence is then cross-correlated with the signal from the 30° receiver. The degree of correlation diminishes as the receiver probe moves away from 30°.

The improvement in the velocity estimate obtained from the use of multiple probes is seen in Fig. 6. The solid line is a simple average of the velocity estimates from 5 individual receivers. The standard deviation of the signal from the 30° receiver is 2.04. The standard deviation for the averaged signal is 1.03. The improvement is a factor of 1.98. If the signals were completely uncorrelated, the improvement would be a factor of $\sqrt{5}$, or 2.24.

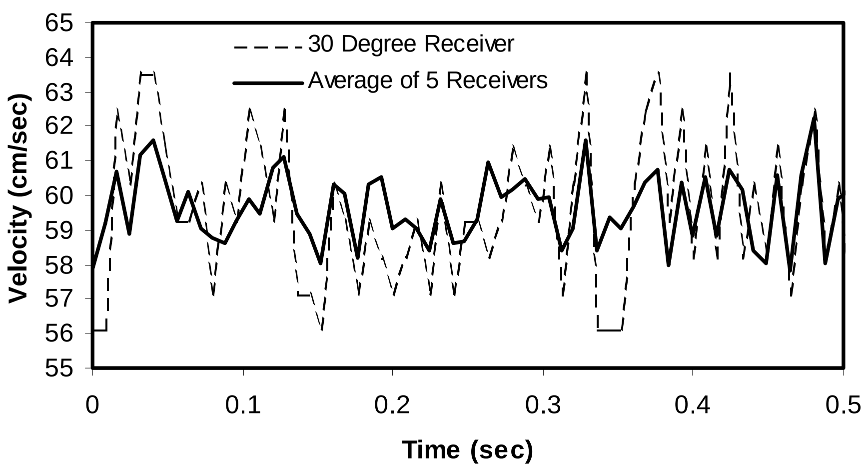


Figure 6: Comparison between velocity estimate from a single receiver (dashed line) and an average of 5 receivers at different angles between 20 and 40 degrees. Use of multiple receivers

Discussion

A simulation was run to determine the relationship between Doppler velocity estimation noise and receiver angle. The simulation included the coherent scattering effects caused by the random configuration of particles and the continuous change in the particle population within the sample volume (Jones and Giddens, 1990). Several assumptions were made to simplify the simulation, and while these may affect the correlation found in Fig. 5 to some degree, they should not affect the overall conclusion of this study, which is that decorrelation occurs within a fairly small angle change.

The geometry was two-dimensional for simplicity. The particle was assumed to undergo a linear trajectory within the plane of the transducers so that spectral broadening as a result of changes in velocity would not affect the results. It is straightforward to extend the simulation to three dimensions and to allow arbitrary particle trajectories. This should be done when complicated flow patterns such as vortex shedding are studied.

The phase of the signal was assumed to be proportional to the pathlength from the center of the transmitter to the particle and then to the center of the receiver. Because the sizes of the transmitter and receiver are finite, the true phase will differ somewhat from the modeled phase, particularly in the near field of each transducer (Morse and Ingard, 1968). However, the simplified relation for phase is sufficient for illustrating the effect of coherent scattering.

This study was performed with a narrow band simulation. It is possible to extend it to wide band processing, and it is not clear at this stage whether the results would be better, meaning a more rapid decorrelation with angle, or worse, meaning a less rapid decorrelation with angle. Because wide band analysis considers frequency components of the returned signal other than the Doppler shift, and because phase depends on wavelength, and hence frequency, the coherent part of the spectrum becomes more complicated in the wide band case. Improvement in velocity estimate for multiple receivers may or may not be independent of that for wide band processing. If the two are independent then an even greater improvement would be obtained if multiple receivers were combined with wide band processing. Otherwise, little improvement would be obtained from the combination.

To ensure that the receivers in this study obtained signals from identical locations, the sample volumes for all receivers were assumed to be identical. Thus, each particle was assumed to contribute the same signal pressure for all probes. In reality, this is not possible unless the sample volume for each receiver is perfectly circular in two dimensions, or spherical in three dimensions, which is unrealistic. In reality, the shape of the sample volume is determined by the transducer beam patterns and the range gating. Had these characteristics been modeled precisely, the sample volumes would have been different for each receiver. Consequently, it would not have been possible to distinguish between improvements in the velocity variance caused by independence of coherent scattering characteristics and those caused by differences in sample volumes; with the receivers having different sample volumes, spatial ambiguity would immediately increase and would naturally lead to a reduction in frequency ambiguity.

Peak tracking was used as the velocity estimator for this study. Since ambiguity is common to all forms of processing, any method should provide similar results. The signals in Fig. 3 demonstrate that the phase differences for probes at different angles are in the signals themselves and not the processing method.

The application of the multiple receiver method to practical applications is subject to several limitations. Two primary difficulties are alignment and access. It is necessary to align each of the receivers and the transmitter such that the sample volumes overlap to the maximum extent possible. Whereas this is relatively easy to do when the propagating medium is homogeneous, it will be more difficult when there are differences in the acoustic impedance of the tissues interceding along the pathway of sound transmission. These differences can be substantial (Goss et al., 1978, 1980) and have been shown to affect Doppler velocimetry in other contexts (Jones et al., 1996; Routh et al., 1987). Also, to investigate the same sample volume from different angles it is necessary to have an acoustic window at each angle. Windows are limited for a number of applications, such as transcraneal (Hames et al., 1991) and cardiac Doppler (Compani et al., 1998).

Another potential limiting factor in the application of this method is the dependence of the scattered intensity on angle. If sound is sufficiently attenuated in the direction of the receiving probe, the signal will not be obtainable at that angle. For the frequencies involved in ultrasound, the highest scattering intensity is in the backscattered direction, toward the transmitter. The ultrasound scattering from blood has been shown theoretically by Angelsen (1980) to involve an angle-independent term that arises from compressibility variations and an angle-dependent term that arises from density variations. When values typical of blood flow are used, the angle-dependent term is sufficiently small to keep the intensity in the direction of minimum scattering above 20% of the backscattered intensity. Measurements by Overbeck et al. (1992) have shown that measurements can readily be taken with a receiver that is oriented 90° from the transmitter.

The final consideration for the multiple receiver method is that the complexity of the signal acquisition instrumentation is increased in proportion to the number of receivers. This becomes less of a problem as electronic technology advances and the cost of complexity in instrumentation decreases. As a result, implementation of multiple receivers in Doppler ultrasound applications should not be prohibited by cost.

Conclusion

The simulation results presented here demonstrate that there is complementary information in the ultrasound signals received at different angles from the scattering particles. This information can be used to improve the estimates of the scatterers' velocity. The method does not violate ambiguity, but rather allows an improved estimate of the underlying Doppler spectrum through an averaging over independent measurements. Whereas there are limitations to the application of this method clinically, potential applications exist, and further investigation of the technique is warranted.

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