I. FLUID MECHANICS

I.1 Basic Concepts & Definitions:

Fluid Mechanics - Study of fluids at rest, in motion, and the effects of fluids on boundaries.

Note: This definition outlines the key topics in the study of fluids:

(1) fluid statics (fluids at rest), (2) momentum and energy analyses (fluids in motion), and (3) viscous effects and all sections considering pressure forces (effects of fluids on boundaries).

Fluid - A substance which **moves** and **deforms continuously** as a result of an **applied shear stress.**

The definition also clearly shows that viscous effects are not considered in the study of fluid statics.

Two important properties in the study of fluid mechanics are:

Pressure and Velocity

These are defined as follows:

Pressure - The **normal stress** on **any** plane through a fluid element **at rest.**

Key Point: The direction of pressure forces will **always** be perpendicular to the surface of interest.

Velocity - The rate of change of position at a point in a flow field. It is used not only to specify flow field characteristics but also to specify flow rate, momentum, and viscous effects for a fluid in motion.

I.4 Dimensions and Units

This text will use both the International System of Units (S.I.) and British Gravitational System (B.G.).

A key feature of both is that neither system uses g_c. Rather, in both systems the combination of units for mass * acceleration yields the unit of force, i.e. Newton's second law yields

S.I. **-** 1 Newton (N) = 1 kg m/s^2 **B.G**. **-** 1 lbf = 1 slug ft/s^2

This will be particularly useful in the following:

Concept	Expression	Units
momentum	$\dot{m}V$	kg/s * m/s = kg m/s^2 = N
		slug/s * ft/s = slug ft/s^2 = lbf
manometry	ρ g h	kg/m^3*m/s^2*m = (kg m/s^2)/ m^2 =N/m^2
		slug/ft^3*ft/s^2*ft = (slug ft/s^2)/ft^2 = lbf/ft^2
dynamic viscosity	μ	N s /m^2 = (kg m/s^2) s /m^2 = kg/m s
		lbf s /ft^2 = (slug ft/s^2) s /ft^2 = slug/ft s

Key Point: In the B.G. system of units, the unit used for mass is the slug and not the lbm. and 1 slug = 32.174 lbm. Therefore, be careful not to use conventional values for fluid density in English units without appropriate conversions, e.g., for water: ρ_w = 62.4 lb/ft^3 **(do not use this value). Instead, use ρ_w = 1.94 slug/ft^3 .**

For a unit system using g_c, the manometer equation would be written as

$$\Delta P = \rho \frac{g}{g_c} h$$

Example:

Given: Pump power requirements are given by

$$\dot{W}_p = \text{fluid density*volume flow rate*g*pump head} = \rho\, Q\, g\, h_p$$

For ρ = 1.928 slug/ft^3, Q = 500 gal/min, and h_p = 70 ft,

Determine: The power required in kW.

$$\dot{W}_p = 1.928 \text{ slug/ft}^3 * 500 \text{ gal/min} * 1 \text{ ft}^3\text{/s} / 448.8 \text{ gpm} * 32.2 \text{ ft/s}^2 * 70 \text{ ft}$$

$$\dot{W}_p = 4841 \text{ ft}^{-}\text{lbf/s} * 1.3558 * 10^{-3} \text{ kW/ft}^{-}\text{lbf/s} = 6.564 \text{ kW}$$

Note: We used the following, 1 lbf = 1 slug ft/s^2, to obtain the desired units

Recommendation: In working with problems with complex or mixed system units, at the start of the problem convert all parameters with units to the base units being used in the problem, e.g. for S.I. problems, convert all parameters to kg, m, & s; for B.G. problems, convert all parameters to slug, ft, & s. Then convert the final answer to the desired final units.

Review examples on unit conversion in the text.

1.5 Properties of the Velocity Field

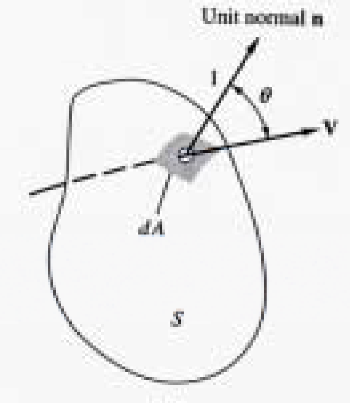
Two important properties in the study of fluid mechanics are:

Pressure and Velocity

The basic definition for velocity has been given previously. However, one of its most important uses in fluid mechanics is to specify both the volume and mass flow rate of a fluid.

Volume flow rate:

$$\dot{Q} = \int_{cs} \overline{V} \cdot \overline{n} \, dA = \int_{cs} V_n \, dA$$



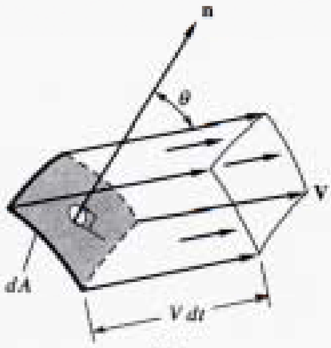
(a)

where V_n is the normal component of velocity at a point on the are a across which fluid flows.

Key Point: Note that only the normal component of velocity contributes to flow rate across a boundary.

Mass flow rate:

$$\dot{m} = \int_{cs} \rho \, \overline{V} \cdot \overline{n} \, dA = \int_{cs} \rho V_n \, dA$$



(b)

NOTE: While not obvious in the basic equation, $\mathbf{V_n}$ must also be measured relative to any motion at the flow area boundary, i.e., if the flow boundary is moving, $\mathbf{V_n}$ is measured **relative to** the moving boundary.

This will be particularly important for problems involving moving control volumes in Ch. III.

1.6 Thermodynamic Properties

All of the usual thermodynamic properties are important in fluid mechanics

P - Pressure	(kPa, psi)
T- Temperature	(^oC, ^oF)
ρ – Density	(kg/m^3, $slug/ft^3$)

<u>Alternatives for density</u>

γ - specific weight = weight per unit volume (N/m^3, lbf/ft^3)

$\gamma = \rho g$ H_2O: $\gamma = 9790\ N/m^3 = 62.4\ lbf/ft^3$

Air: $\gamma = 11.8\ N/m^3 = 0.0752\ lbf/ft^3$

S.G. - specific gravity = ρ / ρ (ref) where ρ(ref) is usually at 4˚C, but some references will use ρ (ref) at 20˚C

liquids ρ (ref) = ρ (water at 1 atm, 4˚C) for liquids = 1000 kg/m^3

gases ρ (ref) = ρ (air at 1 atm, 4˚C) for gases = 1.205 kg/m^3

Example: Determine the static pressure difference indicated by an 18 cm column of fluid (liquid) with a specific gravity of 0.85.

$$\Delta P = \rho g h = S.G.\ \gamma_{ref}\ h = 0.85 * 9790\ N/m^3\ 0.18\ m = 1498\ N/m^2 = 1.5\ kPa$$

Ideal Gas Properties

Gases at low pressures and high temperatures have an equation-of-state (the relationship between pressure, temperature, and density for the gas) that is closely approximated by the ideal gas equation-of-state.

The expressions used for selected properties for substances behaving as an ideal gas are given in the following table.

Ideal Gas Properties and Equations

Property	Value/Equation
1. Equation-of-state	$P = \rho R T$
2. Universal gas constant	$\Lambda = 49{,}700\ ft^2/(s^2\ {}^\circ R) = 8314\ m^2/(s^2\ {}^\circ K)$
3. Gas constant	$R = \Lambda / Mgas$
4. Constant volume specific heat	$C_v = \left.\frac{\partial u}{\partial T}\right)_v = \frac{d u}{d T} = C_v(T) = \frac{R}{k-1}$
5. Internal energy	$d u = Cv(T)\, dT$ u = f(T) only
6. Constant Pressure specific heat	$C_p = \left.\frac{\partial h}{\partial T}\right)_p = \frac{d h}{d T} = C_p(T) = \frac{k R}{k-1}$
7. Enthalpy	$h = u + P v$, $d h = Cp(T)\, dT$ h = f(T) only
8. Specific heat ratio	$k = C_p / C_v = k(T)$

Properties for Air

($R_{air} = 1716\ ft^2/(s^2\ {}^\circ R) = 287\ m^2/(s^2\ {}^\circ K)$)
at 60˚F, 1 atm, $\rho = P/R\,T = 2116/(1716*520) = 0.00237\ slug/ft^3 = 1.22\ kg/m^3$
Mair = 28.97 k = 1.4
$C_v = 4293\ ft^2/(s^2\ {}^\circ R) = 718\ m^2/(s^2\ {}^\circ K)$
$C_p = 6009\ ft^2/(s^2\ {}^\circ R) = 1005\ m^2/(s^2\ {}^\circ K)$

I.7 Transport Properties

Certain transport properties are important as they relate to the diffusion of momentum due to shear stresses. Specifically:

$\mu \equiv$ coefficient of viscosity (dynamic viscosity) $\{M / L\ t\}$

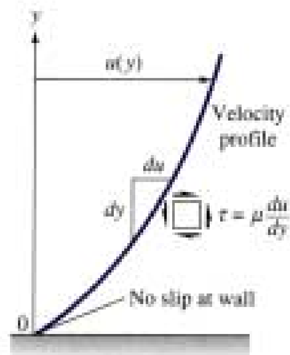
$\nu \equiv$ kinematic viscosity = μ / ρ $\{L^2 / t\}$

This gives rise to the definition of a Newtonian fluid.

Newtonian fluid: A fluid which has a **linear** relationship between shear stress and velocity gradient.

$$\tau = \mu \frac{dU}{dy}$$

The linearity coefficient in the equation is the coefficient of viscosity μ .



Flows constrained by solid surfaces can typically be divided into two regimes:

a. Flow near a bounding surface with
    1. significant velocity gradients
    2. significant shear stresses

This flow region is referred to as a "**boundary layer**."

b. Flows far from bounding surface with
    1. negligible velocity gradients
    2. negligible shear stresses
    3. significant inertia effects

This flow region is referred to as "free stream" or "inviscid flow region."

An important parameter in identifying the characteristics of these flows is the

$$\text{Reynolds number} = \text{Re} = \frac{\rho V L}{\mu}$$

This physically represents the ratio of inertia forces in the flow to viscous forces. For most flows of engineering significance, both the characteristics of the flow and the important effects due to the flow, e.g., drag, pressure drop, aerodynamic loads, etc., are dependent on this parameter.

Surface Tension

Surface tension, Y, is a property important to the description of the interface between two fluids. The dimensions of Y are F/L with units typically expressed as newtons/meter or pounds-force/foot. Two common interfaces are water-air and mercury-air. These interfaces have the following values for surface tension for clean surfaces at 20˚C (68˚F):

$$Y = \begin{cases} 0.0050 \text{ lbf/ft} = 0.073 \text{N/m} \quad \text{air} - \text{water} \\ 0.033 \text{ lbf/ft} = 0.48 \text{ N/m} \quad \text{air} - \text{mercury} \end{cases}$$

Contact Angle

For the case of a liquid interface intersecting a solid surface, the contact angle, θ, is a second important parameter. For θ < 90˚, the liquid is said to 'wet' the surface; for θ > 90˚, this liquid is 'non-wetting.' For example, water does not wet a waxed car surface and instead 'beads' the surface. However, water is extremely wetting to a clean glass surface and is said to 'sheet' the surface.

Liquid Rise in a Capillary Tube

The effect of surface tension, Y, and contact angle, θ , can result in a liquid either rising or falling in a capillary tube. This effect is shown schematically in the Fig. E 1.9 on the following page.

A force balance at the liquid-tube-air interface requires that the weight of the vertical column, h, must equal the vertical component of the surface tension force. Thus:

$$\gamma \pi R^2 h = 2 \pi R Y \cos \theta$$

Solving for h we obtain

$$h = \frac{2 Y \cos\theta}{\gamma R}$$

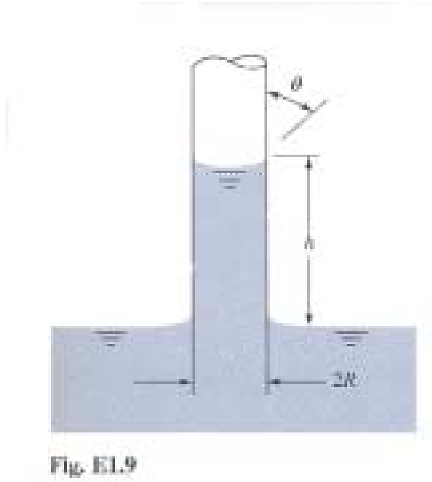


Fig. E 1.9 Capillary Tube Schematic

Thus, the capillary height increases directly with surface tension, Y, and inversely with tube radius, R. The increase, h , is positive for $\theta < 90^\circ$ (wetting liquid) and negative (capillary depression) for $\theta > 90^\circ$ (non-wetting liquid).

Example:

Given a water-air-glass interface ($\theta \approx 0^\circ$, Y = 0.073 N/m, and ρ = 1000 kg/m^3) with R = 1 mm, determine the capillary height, h.

$$h = \frac{2(0.073\, N/m)\cos 0^\circ}{(1000\, kg/m^3)(9.81\, m/s^2)(0.001\, m)} = 1.5\, cm$$

For a mercury-air-glass interface with $\theta = 130^\circ$, Y = 0.48 N/m and ρ = 13,600 kg/m^3, the capillary rise will be

$$h = \frac{2(0.48\, N/m)\cos 130^\circ}{(13{,}600\, kg/m^3)(9.81\, m/s^2)(0.001\, m)} = -0.46\, cm$$