The Uniform Probability Distribution

Bernd Schröder
Definition.

A continuous random variable is said to have uniform distribution on the interval $[A, B]$ if and only if its probability density function is $f(x; A, B) = \begin{cases} \frac{1}{B-A}; & \text{for } A \leq x \leq B, \\ 0; & \text{otherwise}. \end{cases}$

Example. $\text{RAND()}$ is uniformly distributed on $[0, 1]$. 

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**Example.**

\(\text{RAND}()\) is uniformly distributed on \([0, 1]\).
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Example. RAND() is uniformly distributed on \([0, 1]\).
Example.

The random variable $X$ is uniformly distributed on \([2, 5]\). Compute each of the following probabilities.

$$P(X \leq 3) = \frac{3 - 2}{5 - 2} = \frac{1}{3}$$
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Diagram:

2 |--------------------------|-------------------| 5

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![Diagram showing the uniform distribution on [2, 5] with a shaded area representing $P(X \leq 3)$]
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$P(X \geq 4)$

\[
P(X \geq 4) = P(5 \geq X \geq 4) = \frac{5-4}{5-2} = \frac{1}{3}
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Diagram:

```
    2  3.5  5
```

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\[
P(3.5 \leq X \leq 7) = \frac{1}{2 - 3.5} = \frac{1}{0.5} = 2
\]
Example. The random variable $X$ is uniformly distributed on $[2, 5]$. Compute each of the following probabilities.

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Example. *The random variable* $X$ *is uniformly distributed on* $[2, 5]$.
Example. The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$. 

$x = 5 - \frac{0.7}{3} = 2.9$. So $x = 5 - 2.9 = 2.1$. 

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**Example.** The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$. 

\[ x = 5 - \frac{2}{3} = \frac{11}{3} = 3.6666\ldots \]
Example. The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$. 

$$x = 2 \cdot \frac{1}{3} = 0.7 \cdot 3 = 2.1.$$ So $x = 5 - 2.1 = 2.9$. 

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The Uniform Probability Distribution
**Example.** The random variable $X$ is uniformly distributed on $[2,5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$. 

So $x = 5 - 2 \cdot 0.7 = 5 - 1.4 = 3.6$. 

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Uniform Distribution Examples

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The Uniform Probability Distribution
**Example.** The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$. 

![Diagram](image)
Example. The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$. 

$$0.7 = \frac{0.7}{3} = \frac{2.1}{3}$$ 

So $x = 5 - 2.1 = 2.9$. 
Example. The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$. 

\begin{align*}
x &= \frac{0.7 \cdot 3}{5 - 2 - 0.7} \\
&= \frac{2.1}{2.3} \\
&= \boxed{2.9}
\end{align*}
Example. The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$. 

\[
\begin{align*}
0.7 &= \frac{1}{3} \\
\end{align*}
\]
Example. The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$.

\[
\frac{0.7}{1/3} = 0.7 \cdot 3
\]
**Example.** The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$.

\[
\frac{0.7}{1/3} = 0.7 \cdot 3 = 2.1
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**Example.** The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$.

\[
\frac{0.7}{\frac{1}{3}} = 0.7 \cdot 3 = 2.1
\]

So $x = 5 - 2.1$
**Example.** The random variable $X$ is uniformly distributed on $[2, 5]$. Compute the value $x$ so that $P(X \geq x) = 0.7$.

\[
\frac{0.7}{1/3} = 0.7 \cdot 3 = 2.1 \\
\text{So } x = 5 - 2.1 = 2.9.
\]