Discovering Improper Integrals

1. When trying to compute the area under $f(x) = \frac{1}{x^2}$ from $x = 1$ to ∞ we realize that the base has infinite length. But does this mean the area must be infinite? In general this is not the case. As an analogy the sum $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ (ad infinitum) never exceeds 2. (Why?)

2. What can we compute?

   $F(x) := \int_1^x \frac{1}{t^2} dt =$

3. What should the upper bound of the integral be for the area we want to compute?

4. If we want to evaluate a function such as F at a point where the function formally is not defined, what tool do we normally use?

5. Use this tool to find the area.

6. In the same fashion consider the area under $f(x) = \frac{1}{\sqrt{x-3}}$ from 3 to 5. What is "wrong" with this area and how would we compute it?

Some Standard Improper Integrals
(The values of these improper integrals depend on the value of p.)

1. $\int_1^\infty \frac{1}{x^p} dx =$

2. $\int_0^1 \frac{1}{x^p} dx =$

Working with Improper Integrals

1. $\int_1^\infty \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx =$

2. $\int_0^1 \ln(x) dx =$

3. $\int_1^3 \frac{1}{\sqrt[3]{x-2}} dx =$

4. $\int_{-\infty}^\infty x e^{-x^2} dx =$

5. $\int_0^1 \frac{\cos(x)}{x} dx =$