

It took about 15 minutes to generate a book with this table of contents. From experience, I estimate it would take an additional hour to update broken references in Appendix D. From there on, any re-sorting of the modules is a matter of interchanging single lines of code.

This means a fully customized selection can be generated within a day.

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