Linear First Order Differential Equations

Bernd Schröder
What are Linear First Order Differential Equations?

1. A linear first order differential equation is of the form $y' + p(x)y = q(x)$.
2. Recognizing linear first order differential equations requires some pattern recognition.
3. To solve a linear first order differential equation, we turn the left side into the derivative of a product.
   3.1 Compute the integrating factor $\mu(x) = e\int p(x)\,dx$,  
   3.2 Multiply the equation by the integrating factor, note that the left side $e\int p(x)\,dx y' + e\int p(x)\,dx p(x)y$ is the derivative of the product $e\int p(x)\,dx y$, 
   3.3 Integrate both sides, solve for $y$.

That's it.

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   3.3 Integrate both sides, solve for $y$.

That’s it.
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \)
\( y(1) = 0. \)
Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1,$

$y(1) = 0.$

Integrating factor:

$$\int p(x) \, dx$$
Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1$,

$y(1) = 0$.

Integrating factor:

$$\int p(x) \, dx = \int \frac{\cos(x)}{\sin(x)} \, dx$$
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \)
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Integrating factor:
\[
\int p(x) \, dx = \int \frac{\cos(x)}{\sin(x)} \, dx \quad u := \sin(x),
\]
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \)

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$$dx = \frac{du}{\cos(x)}$$
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\int p(x) \, dx = \int \frac{\cos(x)}{\sin(x)} \, dx \quad u := \sin(x), \quad \frac{du}{dx} = \cos(x),
\]
\[
= \int \frac{\cos(x)}{u} \frac{du}{\cos(x)} \quad dx = \frac{du}{\cos(x)}
\]
Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1, \quad y(1) = 0.$

Integrating factor:

$$\int p(x) \, dx = \int \frac{\cos(x)}{\sin(x)} \, dx \quad u := \sin(x), \quad \frac{du}{dx} = \cos(x),$$

$$= \int \frac{\cos(x)}{u} \frac{du}{\cos(x)}$$

$$= \int \frac{1}{u} \, du$$
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$$= \int \frac{\cos(x)}{u} \frac{du}{\cos(x)}$$

$$= \int \frac{1}{u} \, du$$

$$= \ln |u| + c$$
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)}y = 1, \) 
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Integrating factor:
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\int p(x) \, dx = \int \frac{\cos(x)}{\sin(x)} \, dx \quad u := \sin(x), \quad \frac{du}{dx} = \cos(x), \\
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= \int \frac{1}{u} \, du \\
= \ln |u|
\]
Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1,$

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$$= \int \frac{\cos(x)}{u} \, du = \int \frac{1}{u} \, du = \ln|u| = \ln|\sin(x)|$$

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Integrating factor:

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= \int \frac{\cos(x)}{u} \frac{du}{\cos(x)}
\]

\[
= \int \frac{1}{u} \, du
\]

\[
= \ln|u|
\]

\[
= \ln|\sin(x)|
\]

\( \mu(x) = \)
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)}y = 1, \)
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Integrating factor:

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\]

\[
= \int \frac{\cos(x)}{u} \frac{du}{\cos(x)}
= \int \frac{1}{u} \, du
= \ln |u|
= \ln |\sin(x)|
\]

\[
\mu(x) = e^{\int p(x) \, dx}
\]
Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1$, $y(1) = 0$.

Integrating factor:

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\int p(x) \, dx = \int \frac{\cos(x)}{\sin(x)} \, dx \quad u := \sin(x), \quad \frac{du}{dx} = \cos(x),
$$

$$
= \int \frac{\cos(x)}{u} \frac{du}{\cos(x)}
$$

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= \int \frac{1}{u} \, du
$$

$$
= \ln |u|
$$

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= \ln |\sin(x)|
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$$
\mu(x) = e^{\int p(x) \, dx} = e^{\ln |\sin(x)|}
$$
Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)} y = 1$, \[ y(1) = 0. \]

Integrating factor:

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= \ln |u| \\
= \ln |\sin(x)|
\]

\[
\mu(x) = e^{\int p(x) \, dx} = e^{\ln |\sin(x)|} = \sin(x)
\]
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)}y = 1, \)
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Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1$, $y(1) = 0$.

$$\mu(x) \left( y' + \frac{\cos(x)}{\sin(x)}y \right) = 1 \cdot \mu(x)$$
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1 \), \( y(1) = 0 \).

\[
\sin(x) \left( y' + \frac{\cos(x)}{\sin(x)} y \right) = 1 \cdot \sin(x)
\]
Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1$, $y(1) = 0$.

\[
\sin(x) \left( y' + \frac{\cos(x)}{\sin(x)}y \right) = 1 \cdot \sin(x)
\]

\[
\sin(x)y' + \cos(x)y = \sin(x)
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Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)}y = 1, \)
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\sin(x) \left( y' + \frac{\cos(x)}{\sin(x)}y \right) = 1 \cdot \sin(x)
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\sin(x)y' + \cos(x)y = \sin(x)
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\[
\frac{d}{dx} \left( \sin(x)y \right) = \sin(x)
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Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1$, $y(1) = 0$.

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\sin(x) \left( y' + \frac{\cos(x)}{\sin(x)}y \right) = 1 \cdot \sin(x)
\]

\[
\sin(x)y' + \cos(x)y = \sin(x)
\]

\[
\frac{d}{dx} (\sin(x)y) = \sin(x)
\]

\[
\sin(x)y = -\cos(x) + c
\]
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)}y = 1, \) 
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\[
\sin(x) \left( y' + \frac{\cos(x)}{\sin(x)}y \right) = 1 \cdot \sin(x)
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\frac{d}{dx} (\sin(x)y) = \sin(x)
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\sin(x)y = -\cos(x) + c
\]
\[
y = -\frac{\cos(x)}{\sin(x)} + \frac{c}{\sin(x)}
\]
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \)
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\sin(x) \left( y' + \frac{\cos(x)}{\sin(x)} y \right) = 1 \cdot \sin(x)
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\sin(x) y = -\cos(x) + c
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\[
y = -\frac{\cos(x)}{\sin(x)} + \frac{c}{\sin(x)}
\]
\[
= -\cot(x) + \frac{c}{\sin(x)}
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Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \)
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Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1,$

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$$y(x) = -\cot(x) + \frac{c}{\sin(x)}$$
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \) \( y(1) = 0. \)

\[
y(x) = -\cot(x) + \frac{c}{\sin(x)}
\]

\[
0 = y(1)
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Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1,$

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$$y(x) = -\cot(x) + \frac{c}{\sin(x)}$$

$$0 = y(1) = -\cot(1) + \frac{c}{\sin(1)}$$
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \)
\( y(1) = 0. \)

\[
\begin{align*}
y(x) &= -\cot(x) + \frac{c}{\sin(x)} \\
0 &= y(1) = -\cot(1) + \frac{c}{\sin(1)} \\
\frac{c}{\sin(1)} &= \cot(1)
\end{align*}
\]
Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)}y = 1, \)
y(1) = 0.

\[
y(x) = -\cot(x) + \frac{c}{\sin(x)}
\]

\[
0 = y(1) = -\cot(1) + \frac{c}{\sin(1)}
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\frac{c}{\sin(1)} = \cot(1)
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\[
c = \cot(1) \sin(1)
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Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \)
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\[ y(x) = -\cot(x) + \frac{c}{\sin(x)} \]

\[ 0 = y(1) = -\cot(1) + \frac{c}{\sin(1)} \]

\[ \frac{c}{\sin(1)} = \cot(1) \]

\[ c = \cot(1)\sin(1) = \cos(1) \]

\[ y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \]
Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1,$

$y(1) = 0.$

$$y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)}$$
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \) Really Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)}y = 1, \ y(1) = 0? \)
Does $y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)}$ Really Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1$, $y(1) = 0$?

$y' + \frac{\cos(x)}{\sin(x)}y \overset{?}{=} 1$

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Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \)

Really Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)}y = 1, \ y(1) = 0 \)?

\[
\frac{d}{dx} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) + \frac{\cos(x)}{\sin(x)} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) = 1
\]
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \) Really Solve the

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\]

\[
\frac{1}{\sin^2(x)}
\]
Does $y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)}$ Really Solve the Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1, y(1) = 0$?

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\frac{d}{dx} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) + \frac{\cos(x)}{\sin(x)} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) = 1
\]

\[
\frac{1}{\sin^2(x)} - \frac{\cos(1)\cos(x)}{\sin^2(x)} = 1
\]
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \) Really Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)}y = 1, \ y(1) = 0 \)?

\[
\frac{d}{dx} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) + \frac{\cos(x)}{\sin(x)} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) = 1
\]

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\frac{1}{\sin^2(x)} - \frac{\cos(1)\cos(x)}{\sin^2(x)} - \frac{\cos^2(x)}{\sin^2(x)} + \frac{\cos(1)\cos(x)}{\sin^2(x)} = 1
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\]
\[
\frac{1}{\sin^2(x)} - \frac{\cos(1) \cos(x)}{\sin^2(x)} - \frac{\cos^2(x)}{\sin^2(x)} + \frac{\cos(1) \cos(x)}{\sin^2(x)} = 1
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\[
\frac{d}{dx} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) + \frac{\cos(x)}{\sin(x)} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) \equiv 1
\]

\[
\frac{1}{\sin^2(x)} - \frac{\cos(1) \cos(x)}{\sin^2(x)} - \frac{\cos^2(x)}{\sin^2(x)} + \frac{\cos(1) \cos(x)}{\sin^2(x)} \equiv 1
\]

\[
\frac{1 - \cos^2(x)}{\sin^2(x)} \equiv 1
\]
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \) Really Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \ y(1) = 0? \)

\[
\frac{d}{dx} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) + \frac{\cos(x)}{\sin(x)} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) \equiv 1
\]

\[
\frac{1}{\sin^2(x)} - \frac{\cos(1) \cos(x)}{\sin^2(x)} - \frac{\cos^2(x)}{\sin^2(x)} + \frac{\cos(1) \cos(x)}{\sin^2(x)} \equiv 1
\]

\[
\frac{1 - \cos^2(x)}{\sin^2(x)} \equiv 1
\]

\[
\frac{\sin^2(x)}{\sin^2(x)} \equiv 1
\]
Does $y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)}$ Really Solve the
Initial Value Problem $y' + \frac{\cos(x)}{\sin(x)}y = 1, y(1) = 0$?

\[
\frac{d}{dx} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) + \frac{\cos(x)}{\sin(x)} \left( -\cot(x) + \frac{\cos(1)}{\sin(x)} \right) = 1
\]

\[
\frac{1}{\sin^2(x)} - \frac{\cos(1) \cos(x)}{\sin^2(x)} - \frac{\cos^2(x)}{\sin^2(x)} + \frac{\cos(1) \cos(x)}{\sin^2(x)} = 1
\]

\[
\frac{1 - \cos^2(x)}{\sin^2(x)} = 1
\]

\[
\frac{\sin^2(x)}{\sin^2(x)} = 1 \quad \sqrt{\ }
\]
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \) Really Solve the Initial Value Problem
\[ y' + \frac{\cos(x)}{\sin(x)} y = 1, \quad y(1) = 0? \]
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \)\)

Really Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \ y(1) = 0? \)\)

\[ y(1) = \]
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \) Really Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \ y(1) = 0 \)?

\[
y(1) = -\cot(1) + \frac{\cos(1)}{\sin(1)}
\]
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \) Really Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \; y(1) = 0? \)

\[
y(1) = -\cot(1) + \frac{\cos(1)}{\sin(1)} = 0
\]
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \) Really Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \ y(1) = 0 \)?

\[
y(1) = -\cot(1) + \frac{\cos(1)}{\sin(1)} = 0 \quad \sqrt{\ }
\]
Does \( y(x) = -\cot(x) + \frac{\cos(1)}{\sin(x)} \) Really Solve the Initial Value Problem \( y' + \frac{\cos(x)}{\sin(x)} y = 1, \ y(1) = 0 \)?

\[
y(1) = -\cot(1) + \frac{\cos(1)}{\sin(1)} = 0 \quad \checkmark
\]

Yes, it does