Sets and Objects

Bernd Schröder
Two Initial Axioms

Venn Diagrams

Models

Remember Russell’s Paradox
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2. Consequently, we cannot define what “objects” are.
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Terms like “set”, “object” and “belongs to” (or “is an element of”) that remain undefined are called the primitive terms of a theory.
The First Two Axioms

1. There is a set.
2. For every object \( x \) and every set \( S \), we can determine whether \( x \) is an element of \( S \) or not.

Note that the first axiom is similar to the first assumption in Russell's Paradox (but it is not the same). Moreover, the second axiom is exactly the same as the second assumption in Russell's Paradox. Basically, we must make sure that set theory captures the parts that we need without leading to paradoxes.
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Notation and Definitions
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1. Let $x$ be an object and let $S$ be a set. Then we write $x \in S$ if $x$ is an element of $S$. 

2. Let $A, B$ be sets. Then we will say that $A$ is contained in $B$ iff every element of $A$ is also an element of $B$. In this case we will write $A \subseteq B$. If $A$ is not contained in $B$ we will write $A \not\subseteq B$. 

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Visualization With Venn Diagrams
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\[ A \subseteq B \]
Visualization With Venn Diagrams

- A ⊆ B
- C
Visualization With Venn Diagrams

A \subseteq B

C \cap D

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\[ A \subseteq B \]

\[ C \not\subseteq D \]
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Visualization With Venn Diagrams

$A \subseteq B$

$C \not\subseteq D$

$E \not\subseteq D$
How Do We Know that the Axioms Do Not Lead to a Contradiction?

A model for a set of axioms is a way to assign meanings to the primitive terms so that all the axioms become true statements. A model does not define what the primitive terms are in general. It merely specifies a situation in which the primitive terms “work”. There can be more than one model for a (consistent) set of axioms.
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Models for the Axioms
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\[ S \]

\[ a \quad e \]

\[ b \quad c \quad d \]
Models for the Axioms

\[ S \]

\[ a \quad e \quad f \]
\[ b \quad c \quad d \]

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Set $S$ with elements $a$, $b$, $c$, $d$, and $e$, and an element $f$. The set $C$ contains $c$ and $d$.
Models for the Axioms

\[ S \]

- \( a \)
- \( b \)
- \( c \)
- \( d \)
- \( e \)
- \( f \)

\[ V \]

\[ C \]
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