Technical Report

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Selection of a Lens for Night and Day Balloon Observing and Automated Star Identification

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I. Introduction

The MARGIE (Minute of Arc Resolution Gamma-ray Imaging Experiment) Group at LaTech is developing an attitude determination system for use with the MARGIE telescope being developed by groups at the University of New Hampshire and Louisiana State University. Accurate position determination on the celestial sphere is based on identification of stars by cameras that are fixed relative to the primary detector. Algorithms for this purpose were developed for the Hubble Telescope, and have been used by all subsequent astrophysical satellites. The pattern recognition depends on knowing the angular separations between star centroids accurately, and the apparent magnitudes moderately accurately.

Balloon groups have been flying CCD cameras for about 10 years, to determine absolute attitudes for x-ray and γ-ray detectors. So far, these groups have been able to determine moderately accurate positions at night, but have not been able to by day for a variety of reasons. The cameras must be angled sufficiently far from the balloon to avoid scattered sunlight, but look through as little atmosphere as possible, since even at 120,000 feet, the effect of scattered sunlight is considerable [Gehrels, Barthelmy]. Using a lens with a long focal length and small field of view (FOV) allows one to obtain maximum angular resolution, but when sunlight decreases the limiting magnitude that can be reached, there are no longer enough bright stars available [de Bernadis *et al.*]. On the other hand, using a wider angle lens not only reduces the angular resolution, but since such a lens usually as a smaller aperture, it decreases the amount of light that can be gathered from a star in a fixed exposure time. In order for a camera to be useful on a balloon day and night, it needs to pick up enough bright stars, and to be fast enough to be able to operate on a short exposure time. This is necessary because the motion of the balloon causes trailing of the star images, and this decreases the signal to noise ration. Star identification algorithms, such as those developed for the Hubble Telescope, and used on all later astrophysical satellite missions, require accurate centroid separations, and moderately accurate apparent magnitudes. Extensive calculations have been carried out to determine whether it is feasible to satisfy all the requirements during the day as well as at night. This does appear to be the case. either a 55 mm or a 50 mm lens of sufficient aperture can be used. The 55 mm f/1.2 lens would be the best option, but it is no longer in production, and has to be obtained as a used lens when available. the best option in regular production would be a 50 mm f/1.2 lens.

II. Camera Parameters and Lenses Considered

1. *CCD Chip*

The AP1 camera has a CCD chip with the following specifications.

Dimensions: 6.9 mm by 4.6 mm
Number of pixels: 768 x 512
Pixel size: 9μ x 9μ
Dark current: 10 pA/cm^2 at 25 °C, which is equivalent to 50.5 e$^-$ /pixel at 25 °C,
or 1.6 e$^-$ /pixel at an operating temperature of -5 °C

2. *Lenses Considered*

The 92 mm lens in the Rossi XTE Ball star trackers has been included in all the calculations for comparison with the other lenses being considered. Calculations have been carried out for the following lenses.

Table 1. Lenses Considered for Day and Night Viewing

Focal Length *mm*	f-ratio	Comments
92	f/1.75	Rossi XTE lens
135	f/2.8	Our first lens, chosen to give high angular resolution
90	f/1.8	Close to the focal length chosen for the Rossi XTE
85	f/1.4	Wider aperture than the standard 90 mm lens. Very expensive.
55	f/1.2	Wide FOV, giving larger number of bright stars for daytime observing. No longer available new - replaced by 50 mm f/1.2.
50	f/1.4	Focal length is that of the normal standard lens sold for 35 mm cameras
50	f/1.2	Wider aperture, gathering more light for daytime viewing

III. Calculations Related to Lens Performance

1. *Image Size*

The angle A on the sky corresponding to each side of the chip is given by

$$\tan A/2 = s/2f,$$

where s is the length of a side of the chip, and f is the focal length of the lens in the same units.

2. *Field of View*

The field of view and the image scale of the chip are given for a number of lenses in the following table.

Table 2. Field of View and Image Scale of Chip for a Number of Lenses

Lens Type	Focal Length *mm*	Focal Ratio	Chip Field of View Length *deg*	 Width *deg*	 Area *deg*^{2}	Image Scale, I_s *″/pix*	
Rossi XTE	92	f/1.75	8	8	64	56.3	CT-601's chip
Telephoto	135	f/2.8	2.9	1.95	5.7	13.7)	
	90	f/1.8	4.4	2.9	12.9	20.6)	with AP1's chip:
	55	f/1.2	7.2	4.8	34.4	33.6)	KAF-0400
Standard	50	f/1.4 or f/1.2	7.9	5.3	41.9	37.0)	

3. ***Movement of Stars Across the Chip and Exposure Times***

Ball Aerospace tests have shown [Proposal for XTE Star Tracker] that the CT-601 solid-state star tracker supplied to NASA for the Rossi XTE satellite is capable of maintaining the specified accuracy in magnitude and position: intensity error ≤ 20%, centroiding error < 2", and a noise equivalent angle (NEA) due to random error in background, readout noise, etc. < 5.5" for a 6th magnitude star, 2.3" for 5^m for angular speeds of a star relative to the camera up to 0.3 degrees per second. After that, the SNR decreases and the chip, which has 512 x 512 pixels, 35μ per pixel, and lens focused so that a stationary star has a diameter of two pixels, movement of 0.3° per second amounts to displacing the centroid by two pixels during a 100 ms exposure. This displacement is used to calculate upper limits for acceptable exposure times for lenses that may be used with the Apogee AP1 chip.

The maximum angular rate of a star observed from a balloon is expected to around $0.2° \, s^{-1}$ on the basis of communications from Scot Barthelmy (NASA) and Robert Lockwood (LSU) that the rate of rotation of a balloon is expected to be no more than 1 revolution per hour, and the pendulum angle $\sim 1/2° \, s^{-1}$ with a period of 16-18 s, once the balloon has stabilized. The maximum exposure times, calculated according to the criteria used for the CT-601 camera, are shown in Table 3.

Table 3. Maximum Exposure Times for Lenses Used with the AP1 Camera's Chip

Lens Type	Focal Length *mm*	Maximum Exposure Time *seconds*
Rossi XTE	92	0.104 CT-601's chip
Telephoto	135	0.035)
	90	0.035) AP1's chip:
	55	0.085) KAF-0400
	50	0.093)

These times are calculated using the image scales from Table 2, and the formulas

$$\theta' = \Delta\theta/\Delta t = 2\,I_s/\Delta t \quad \Rightarrow \quad \Delta t = 2\,I_s/\theta'.$$

4. ***Number of Stars in Field of View***

Another factor that is of primary importance in choosing a lens is the number of stars expected in the field of view. Charts in Ball's *Proposal for the XTE Star Tracker* indicate that the relations between log N and m , where N is the average number of stars per square degree and m is the apparent magnitude, are approximately linear, and may be calculated using

$\log N = 0.5m - 4$ for averages over the whole celestial sphere, or

$\log N = 0.5m - 3.7$ within 10° of the Galactic poles.

Numbers expected for each of the lens-chip combinations being considered are shown in Table 4.

Table 4. Numbers of Stars in the Field of View for Each Lens-Chip Combination

A. *Numbers using star density near the galactic poles*

Focal Length *mm*	FOV Area *deg^2*	5.5	6	6.5	7	7.5	8	8.5	9	Apparent magnitude
		0.056	0.1	0.18	0.31	0.56	1	1.8	3.2	N near Galactic poles
92	64	3.6	6.4	11.5	19.8	35.8	64	113.4	205	No. in FOV of each lens to a given apparent magnitude
135	5.7	0.3	0.6	1.0	1.8	3.2	5.7	10.2	18	
90	12.9	0.7	1.3	2.3	4.0	7.2	12.9	22.9	41	
55	34.4	1.8	3.2	5.8	10.7	19.3	32.1	61.2	110	
50	41.6	2.3	4.3	7.5	12.9	23.3	41.6	74.0	133	

Table 4.

B. *Numbers using the average star density over the celestial sphere*

Focal Length *mm*	FOV Area *deg^2*	5.5	6	6.5	7	7.5	8	8.5	9	Apparent magnitude
		0.11	0.2	0.36	0.62	1.26	2	3.6	6.4	All-sky Average N
92	64	7.2	12.8	23.0	39.7	71.7	128.0	227.6	410	No. in FOV of each lens to a given apparent magnitude
135	5.7	0.6	1.1	2.1	3.5	6.4	11.4	20.3	37	
90	12.9	1.4	2.6	4.6	8.0	14.4	25.7	45.8	82	
55	34.4	3.6	6.4	11.6	21.3	38.5	64.2	122.3	220	
50	41.6	4.7	8.3	15.0	25.8	46.6	83.3	148.0	266	

The ideal number of stars down to a chosen apparent magnitude is between three and five. While in principal star identification can be performed using sufficiently accurate information from just two stars [Proposal for XTE Star Tracker], in practice such accuracy may be difficult to achieve, and the data would have to be cleaned of cosmic rays first. The risk of misidentification is such that NASA's MTASS (Multi-mission Three-Axis Stabilized Spacecraft) attitude software requires a minimum of three stars in a frame. A maximum of five is used, because this is the maximum number that can be selected for tracking, and has also been shown to be sufficient for a very high degree of reliability [CSC: Simulations for the XTE].

5. *Sky Brightening on Balloon Flights*

One problem that the satellite star tracker does not have to deal with, but which we must on a balloon, is daytime brightening of the sky background. this is equivalent to a two-magnitude degradation in the stars that can be observed in a given exposure time, which means that a balloon must have enough stars available for limiting magnitudes spanning a two-magnitude interval.

6. *Time Required to Observe Stars of Given Brightness with 20% Uncertainty*

The next step is to calculate how long it will take the AP1's KAF-0400 chip to collect enough light from a star of a given apparent magnitude with each lens to determine the star's intensity to 20% accuracy. This means that the SNR needs to be around 5 [Mallama]. The formulas used to calculate exposure time may be found in Mallama or Rybski and are summarized by Guzik, who also gives estimates of atmospheric transparency (and seeing) for the Baton Rouge area. The formulas and estimate are included in the Appendix.

Parameters used in calculating these times are

Dark current	1.4 electrons/pixel/sec
Quantum efficiency	0.35 at yellow wavelengths
Readout noise	15 electrons/pixel (AP1 specs: 10-15 e$^-$ /pixel)
Transmission efficiency of lens	0.95 (This was based on the maximum thickness of the lens, and the measured window absorption.)
Transmission efficiency of window	0.80 at yellow wavelengths
Bandwidth	1000 Å (No filter will be used, but the quantum efficiency of the chip drops at short wavelengths.)
Atmospheric transmission	0.99 assumed for 20 mile altitude

An ideal image of a star stationary relative to the camera would cover 4 pixels. (The image is normally defocused slightly to allow sub-pixel accuracy on the centroiding.) Assuming that the star's centroid moves two pixels during an observation, the final image will cover 8 pixels. Results of the calculations for the lenses under consideration are summarized in Table 5. Part A gives results with no window in front of the camera, and part B gives results with a proposed window absorbing 20% of incident yellow light.

Table 5. Time in Seconds to Reach an Apparent Magnitude with Error in Intensity ≤ 20%
A. *No window*

Focal Length *mm*	Apparent Magnitude								
	5.5	6	6.5	7	7.5	8	8.5	9	
92	0.004	0.007	0.011	0.017	0.028	0.045	0.073	0.122	Time in seconds
135	0.005	0.008	0.013	0.021	0.033	0.053	0.083	0.133	
90	0.005	0.008	0.013	0.021	0.033	0.053	0.083	0.034	
55	0.006	0.010	0.016	0.025	0.040	0.063	0.101	0.164	
50, f/1.2	0.008	0.012	0.019	0.030	0.048	0.077	0.123`	0.201	
50, f/1.4	0.010	0.016	0.025	0.040	0.065	0.104	0.166	0.273	

Table 5. B. *With window*

Focal Length *mm*	Apparent Magnitude								
	5.5	6	6.5	7	7.5	8	8.5	9	
92	0.005	0.007	0.012	0.019	0.030	0.047	0.074	0.124	Time in seconds
135	0.006	0.009	0.015	0.023	0.036	0.058	0.091	0.146	
90	0.006	0.009	0.015	0.023	0.036	0.058	0.091	0.046	
55	0.007	0.011	0.017	0.027	0.043	0.069	0.108	0.173	
50, f/1.2	0.008	0.013	0.021	0.033	0.053	0.083	0.131	0.210	
50, f/1.4	0.011	0.018	0.028	0.044	0.070	0.111	0.175	0.281	

IV. Lenses Showing Enough Stars in Acceptable Exposure Time

Comparing Tables 3-5, we can see that the 135 mm f/2.8 lens is able to see enough stars at low to moderate galactic latitudes at 7th magnitude, in the time that it would take a star traveling at the maximum expected velocity to move two pixels. However, its field of view is too small for it to pick up enough stars at high galactic latitudes, and if the assumed sky background of 20^m is appropriate for nighttime observing, it would not see enough star at 5^m in the daytime. A 90 mm f/1.8 lens would pick up a much better number of stars for a nighttime flight, but would again be in trouble during the day. A 55 mm f/1.2 lens would be able to see enough stars day or night at low galactic latitudes, and would have a 62% probability of seeing at least three $6^m.5$ stars near the galactic poles during the day. This probability was calculated using a formula from Ball's XTE Proposal, based on the Poisson distribution. the probability of seeing at least N stars is

$P_N = 1 - \Sigma e^{-a} a^k /k!$ where the summation runs from 0 to N-1,

and 'a' is the average number of stars for the FOV and a given apparent magnitude. The probability that a 50 mm f/1.2 lens is 79%. The average numbers of stars seen during a 100 ms exposure by 55 mm and 50 mm f/1.2 lenses are shown in Table 6.

Table 6. Average Number of Stars Seen by Two f/1.2 Lenses in 100 ms

Lens Focal Length	Apparent Magnitude		
	6	8	
55 mm	6.4	64.2	All-sky Average
	3.2	32.1	Near Galactic Poles
50 mm	8.3	83.2	All-skyAverage
	4.2	41.6	Near Galactic Poles

V. Other considerations

1. *Uncertainties Due to Color*

Light from a blue star suffers more refraction at a window or lens than a red star. At high galactic latitude, there is a much higher proportion of blue stars than near the galactic plane, but we have no *a priori* knowledge of the colors of stars in any frame being processed. The CT-601 camera is sealed in a container with a fused quartz window, but one of NASA's specifications was that the excess color deviation of a blue over a red image must be no more than 2µ and has been measured at 6.1µ. This corresponds to a 3" error, which becomes an irreducible part of the error budget. The small error was accomplished by using a color-corrected, i.e. composite, lens. For the engineering flight, the error due to color using a window and a simple lens will simply be part of our error budget. the amount of the error will be measured in the laboratory using LEDs when the final lens is purchased. Its effect will be more serious at high galactic latitudes, where there is a higher proportion of blue stars, and fewer stars are available.

The 1/4 inch window also introduces uncertainty into the magnitude estimates. The maximum angle of incidence of starlight on the lens is half the FOV angle, which depends on the focal length of the lens. Using a 135 mm lens with 3° x 2° FOV and photographing the light from a blue LED fed through six fiberoptic cables, we found that the absorption of light from the central fibers was 20%, but from the outer fibers was 40%. Changing the angle of incidence from roughly 0° to 1/2° apparently doubled the absorption. Using a 55 mm lens, the angle of incidence could be up to 3.6°, increasing both the error in estimating the magnitude, and the positional error.

2. *Availability and Price*

The 55 mm f/1.2 lens is no longer in production, and is available only as a used lens one from a camera exchange. The other camera lenses described are in regular production. The prices vary enormously. Used lenses in good condition are typically priced between $50 and $150. 50 mm f/1.4 lenses are frequently available used, but a 50 mm f/1.2 lens would probably need to be special-ordered at a price of around $580.

VI. Recommendations

Picking up enough stars over a two-magnitude interval within the time limit appropriate for each lens is essential if that lens is to be used for night and day balloon observing. This requires a moderately wide field of view. Both the 55 mm and the 50 mm lenses have a sufficiently wide field of view to pick enough bright stars by day. If the lenses are also going to pick up star images within the time that it will take a centroid to move two pixels - around 100 ms for these focal lengths - they need wide aperture. The 55 mm lens can reach $8^m.5$ with 20% uncertainty in 100 ms if there is no window in front, or 110 ms with a window. The 50 mm f/1.2 lens will be able to reach a magnitude of ~ 8.3 without a window in 100 ms, and would need a little longer with a window. The 50 mm f/1.4 lens would reach 8^m with a window in 110 ms. Either the 55 mm f/1.2 or the 50 mm f/1.2 lens would be useable. The latter is preferable for the number of stars available in any part of the sky, but is very expensive - the shift from production of 55 mm to 50 mm at f/1.2 is too recent for used lenses to be available. The 50 mm f/1.4 lens could be used on an engineering flight, with the understanding that at high galactic latitudes there would often not be enough stars except during the dark hours.

References

Ball Aerospace Electro-Optics/Cryogenics Division. *CT-601 Solid State Star Tracker*.

Ball Aerospace P92-483. *Proposal for XTE Star Tracker*. 1992.

Barthelmy, S. NASA/Goddard Space Flight Center, Gamma Ray and Cosmic Ray Astrophysics Branch, Engineer. *Personal communications.*

Computer Sciences Corporation CSC/TR-92/6002R0UD0
NASA/Goddard 554-FDD-92/001R0UD0
Fixed Head Star Tracker (FHST) Performance Study.

de Bernadis, P. *et al.* "The ARGO Balloon Telescope: A Cosmology Research Tool". In *Observational Cosmology,* ASP Conference Series, v. 51, 1993, 519.

Gehrels, N. NASA GSFC, Head, Gamma Ray and Cosmic Ray Astrophysics Branch. *Personal communications.*

Guzik, G. Baton Rouge Observatory Technical Report 97-002. *A Comparison of Potential CCD Cameras for Use at the Observatory.* 1997. (Online at http://phys.lsu.edu/observatory/tchrpt9702.html)

Mallama, A. "The Limiting Magnitude of a CCD Camera". Sky & Telescope, February 1993.

Rybski, P. M. "What Can You Really Get from Your CCD Camera?". CCD Astronomy, Summer 1996.

APPENDIX

Formulas Used To Calculate Exposure Times

Top of Atmosphere Flux K from a star of magnitude *m* is given by

$$K = 10^{(15-2m)/5} \qquad \text{[Guzik]}$$

Transmitted Light

Let A_L be the effective aperture of the lens in cm^2
R_L be the transmission efficiency of the lens
B be the bandwidth in Angstroms
QE be the quantum efficiency of the CCD at a given wavelength
and T_{Atm} be the transmission efficiency of the atmosphere.

Then the light received from a star in electrons/second is given by

$$s = A_L \, R_L \, B \, QE \, T_{Atm} \, K$$

Parameter Values Used

A_L = 16.5 cm^2 for a 55 mm f/1.2 lens, or 13.6 cm^2 for a 50 mm f/1.2 lens.

$R_L = 0.97$ at yellow wavelengths, estimated from the maximum thickness of the lens, and using the measured absorption of possible windows

B = 1000 Å No filter will be used, but the quantum efficiency of a chip decreases at short wavelengths. the calibration A0V stars peak at ~ 5800 Å.

QE = 0.35` for a front-lit chip at $\lambda \sim 6000$ Å

$T_{Atm} = 0.75$ This figure was assumed by Guzik for the Baton Rouge Observatory. The Ruston/Simsboro area is not likely to have better transparency.

Relative Uncertainty

The relative uncertainty U is determined by

$$U = \sigma_s / S = \sqrt{}(S+2Bg)/S, \qquad \text{[Guzik]}$$

where S = *st*, *s* being the number of electrons per second generated by a star (see below), *t* is the time in seconds, and Bg is the number of background electrons generated per second by the sky background and camera noise.

B is calculated from

$$B = n(b+d)t + nR^2.$$

Here n is the number of pixels collecting starlight, ideally four in this work, b is the number of electrons per pixel per second generated by the sky background, and R is the readout noise in electrons per pixel.

Sky Brightness

Rybski gives $18^m.5$ as a typical value for a moonless night sky in a rural area 30 miles from a city. Guzik considers 17^m more likely at the Baton Rouge Observatory site on the southern outskirts of Baton Rouge. This would probably be appropriate for LaTech's observatory at Simsboro. Below, and even at, this level the sky background will be negligible compared with our readout noise, given the shortness of the exposures.

Calculation of Exposure Times

From the formula for relative uncertainty, a quadratic equation for *t* may be obtained:

$$U^2 s^2 t^2 - [s + 2n(b+d)]t - 2nR = 0.$$

solve for *t*, taking the + sign before the radical.

ADC Counts Expected for Each Apparent Magnitude

The AP1 has a well depth of ~ 85,000 e^- /pixel, and LaTech's camera has 12-bit digitization, giving a maximum count of 4092. The gain is set at 32 for this work. To convert apparent magnitude to counts per pixel per 0.1 second from starlight, use

$$\text{ADC counts} = s(m)\cdot 32\cdot(4/85)\cdot 0.1,$$

For example, using the 55 mm f/1.2 lens, we would have

m	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
Counts	4118	2598	1639	1034	653	411	261	163