Chapters 7 and 8
Rate of Return Analysis
Incremental Rate of Return Analysis

1. (Internal) Rate of Return
   - The interest rate that makes any of the following true:
     - PW Benefits – PW Costs = 0
     - PW Benefits/PW Costs = 1
     - EUAB – EUAC = 0
     - EUAB/EUAC = 1

   *Notice that each of the above represents the same concept, only in different forms.

2. Calculating an Alternative’s ROR
   - Step 1: Choose one of the equations listed in note #1.
   - Step 2: Write out the appropriate equation based on the alternative’s cash flows.
   - Step 3: Use trial and error to find the ___________________________ that makes the ___________________________. Linear interpolation (discussed later) will often be needed to find the exact interest rate.

   - Note: ROR is calculated on an annual basis unless otherwise state.

3. An Example
   - Given the following cash flows, determine the ROR.

<table>
<thead>
<tr>
<th>EOY</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2250</td>
</tr>
<tr>
<td>1</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>800</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
</tr>
</tbody>
</table>
• We know that the ROR is between ____% and ____%. Therefore, we need to use linear interpolation to find the exact rate.

• While we could use formulas (for the factors) to find the exact rate of return, we will usually save time by interpolating between the two interest rates (from the back of the book) that give us the closest (to zero) positive NPW and closest (to zero) negative NPW.

***************Interpolation on NPW (or EUAB – EUAC) Values***************

ROR =

lower % + [(higher % - lower %) * \left( \frac{NPW \text{ at lower } \% - 0}{NPW \text{ at lower } \% - NPW \text{ at higher } \%} \right)]

*Note: For EUAB – EUAC values, substitute EUAB – EUAC wherever NPW appears in the above equation.*

Therefore,

ROR =
4. Another Example

- Find the ROR for the following alternative.

<table>
<thead>
<tr>
<th>EOY</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1000</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
</tr>
<tr>
<td>4</td>
<td>750</td>
</tr>
</tbody>
</table>

- In this situation, we only have one unknown factor (i.e., P/G, i%, 4).

- In situations like this, it is easier to solve for the unknown factor and search for its value in the interest tables in the back of the book.

- We know the ROR is between ___% and ___%. Therefore, we will use linear interpolation to find the exact ROR.

- The linear interpolation is similar to that used earlier.

**********************************Interpolation on Factor Values (Note: This only works for situations with one unknown factor)**********************************

\[
\text{ROR} = \text{lower }\% + \left[ (\text{higher }\% - \text{lower }\%) \times \frac{\text{Factor Value at lower }\% - \text{Factor Value at ROR }\%}{\text{Factor Value at lower }\% - \text{Factor Value at higher }\%} \right]
\]

**********************************
Therefore,

\[ \text{ROR} = \]

5. Making a Decision: One Alternative
- If the ROR \( \geq \) MARR, the alternative is acceptable.
- If the ROR < MARR and *do-nothing* is an option, *do-nothing* should be chosen.
- If the ROR < MARR and *do-nothing* is NOT an option, then the alternative is to be chosen because it is the only available alternative.

6. Incremental ROR (\( \Delta \text{ROR} \)) Analysis
- Incremental ROR analysis *must* be used when choosing between ______________________.
- Incremental ROR is the ROR that is found on the ______________________
  ______________________.
- The difference is calculated by subtracting the cash flows for the lower cost alternative from the cash flows for the higher cost alternative (i.e., high cost alternative – low cost alternative)
- The high/low cost alternative is based on the higher/lower initial cost. If you were to have the EUAB and EUAC calculations for both alternatives, then use EUAC to determine the higher/lower cost alternative.
7. Making a Decision: Two Alternatives
   • If $\Delta ROR > MARR$, choose the ___________________________.
   • If $\Delta ROR < MARR$, choose the ___________________________.

8. Example
   • Use incremental ROR ($\Delta ROR$) Analysis to choose between the following two alternatives.
   • $MARR = 9\%$.

\[
\begin{array}{|c|c|c|}
\hline
\text{EOY} & \text{Alt. A} & \text{Alt. B} \\
\hline
0 & -17500 & -21000 \\
1 & 5000 & 5000 \\
2 & 5000 & 5500 \\
3 & 5000 & 6000 \\
4 & 5000 & 6500 \\
5 & 5000 & 7000 \\
\hline
\text{ROR} & 13.20\% & 12.40\% \\
\hline
\end{array}
\]

   • Note: Both options are acceptable based on the MARR.
   • Since Alt. B is the higher cost alternative, we will calculate the incremental cash flow based on _______________________

\[
\begin{array}{|c|c|c|}
\hline
\text{EOY} & \text{Alt. A} & \text{Alt. B} \\
\hline
0 & -17500 & -21000 \\
1 & 5000 & 5000 \\
2 & 5000 & 5500 \\
3 & 5000 & 6000 \\
4 & 5000 & 6500 \\
5 & 5000 & 7000 \\
\hline
\text{ROR} & 13.20\% & 12.40\% \\
\hline
\end{array}
\]
9. Another Example

- Utilize ROR Analysis to decide which, if any, of the following alternatives should be accepted.
- MARR = 10%.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>$2,000</td>
<td>$3,000</td>
</tr>
<tr>
<td>Annual Benefits</td>
<td>$700</td>
<td>$1,100</td>
</tr>
<tr>
<td>Annual Costs</td>
<td>$100</td>
<td>$300</td>
</tr>
<tr>
<td>Salvage Value</td>
<td>$500</td>
<td>$750</td>
</tr>
<tr>
<td>Useful Life</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

- The first thing we need to do is to make sure we have at least one alternative with a ROR ≥ MARR.

- Alternative #1
- \( \text{NPW} = 0 = -2000 + 700(\text{P/A, i\%, 5}) - 100(\text{P/A, i\%, 5}) + 500(\text{P/F, i\%, 5}) \)

  - At 15%
  - \( \text{NPW} = -2000 + 700(3.352) - 100(3.352) + 500(.4972) \)
  - \( \text{NPW} = +259.80 \)

  - At 18%
  - \( \text{NPW} = -2000 + 700(3.127) - 100(3.127) + 500(.4371) \)
  - \( \text{NPW} = +94.75 \)

  - At 20%
  - \( \text{NPW} = -2000 + 700(2.991) - 100(2.991) + 500(.3277) \)
  - \( \text{NPW} = -41.55 \)

- We know that the ROR is between 18% and 20%. Therefore, we will use linear interpolation to find the exact ROR.
ROR for Alternative #1

ROR =

\[ 18\% + \left[ (20\% - 18\%) \times \left( \frac{94.75 - 0}{94.75 - 41.55} \right) \right] \]

= 18\% + 2\%(0.695)

= 19.39\%

- Because we know that at least one of the alternatives is acceptable, the do-nothing option is eliminated, and we can move on to Incremental Analysis. Remember to use high cost – low cost.
10. Another Example
- Given the following two alternatives and Rate of Return Analysis, determine which, if any, should be chosen.
- MARR = 15%

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Classic</th>
<th>Platinum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Cost</td>
<td>$20,000</td>
<td>$33,000</td>
</tr>
<tr>
<td>Annual Benefits</td>
<td>$7,000</td>
<td>$8,500</td>
</tr>
<tr>
<td>Annual Costs</td>
<td>$1,000</td>
<td>$1,500</td>
</tr>
<tr>
<td>Salvage Value</td>
<td>$2,000</td>
<td>$2,500</td>
</tr>
<tr>
<td>Useful Life</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

- The first thing we need to do is to make sure that at least one alternative has a ROR \( \geq \) MARR.

- Alternative: Classic
- \( NPW = -20000 + 6000(P/A, i\%, 5) + 2000(P/F, i\%, 5) \)
Because we know that at least one of the alternatives is acceptable, the do-nothing option is eliminated, and we can move on to Incremental Analysis. Remember to use high cost – low cost.

<table>
<thead>
<tr>
<th>EOY</th>
<th>Classic</th>
<th>Platinum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-33000</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7000</td>
<td></td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>9500</td>
<td></td>
</tr>
</tbody>
</table>