Engineering Economy
Chapter 4
More Interest Formulas

1. Uniform Series Factors Used to Move Money
- Find F, Given A (i.e., F/A)
- Find A, Given F (i.e., A/F)
- Find P, Given A (i.e., P/A)
- Find A, Given P (i.e., A/P)

2. What is a Uniform Series?
- A series of payments that is ________________________________.
- A series of payments that is _________________________.
- A uniform series is denoted with the letter *A*.

3. Find F, Given A (F/A, i%, n)
- This factor finds an equivalent single payment (i.e., F) from a uniform series of payments (i.e., A).
- *F* is a point in time located *exactly* where the last uniform payment occurs.
- *A* is a uniform series of points in time where the uniform amounts of money are currently located.
- *i%* is the effective interest rate for the period of time between any two consecutive uniform payments.
- *n* is the number of uniform payments that are moved forward. *Note: This includes the last payment where F is located.*

4. Find A, Given F (A/F, i%, n)
- This factor finds an equivalent uniform series of payments (i.e., A) from a single payment (i.e., F).
- *A* is a uniform series of points in time that end *exactly* where a single amount of money is currently located.
- *F* is a point in time where the money is currently located.
- *i%* is the effective interest rate for the period of time between any two consecutive uniform payments.
- *n* is the number of equivalent uniform payments that are found from the single payment (i.e., F).

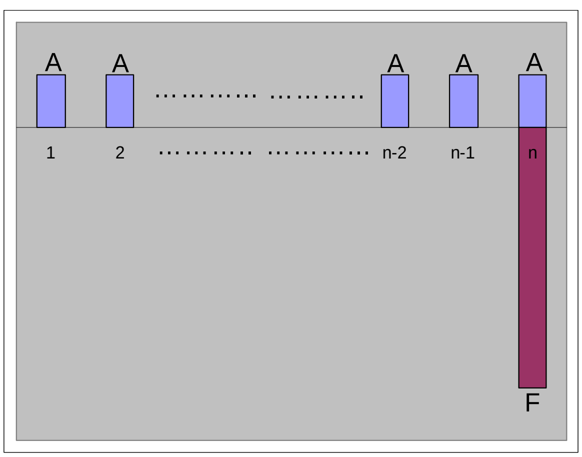
5. Find P, Given A (P/A, i%, n)

- This factor finds an equivalent single payment (i.e., P) from a uniform series of payments (i.e., A).
- *P* is a point in time *one period* prior to where the first uniform payment occurs.
- *A* is a uniform series of points in time where the uniform amounts of money are currently located.
- *i%* is the effective interest rate for the period of time between any two consecutive uniform payments.
- *n* is the number of uniform payments that are moved backwards.

6. Find A, Given P (A/P, i%, n)

- This factor finds an equivalent uniform series of payments (i.e., A) from a single payment (i.e., P).
- *A* is a uniform series of points in time that begin *one period* after where a single amount of money is currently located.
- *P* is a point in time where the money is currently located.
- *i%* is the effective interest rate for the period of time between any two consecutive uniform payments.
- *n* is the number of equivalent uniform payments that are found from the single payment (i.e., P).

7. Derivation of the F/A Equation



- Use the F/P equation to solve for F
- $F = A(1+i)^{n-1} + A(1+i)^{n-2} + \ldots + A(1+i)^2 + A(1+i)^1 + A$
- Multiply above by (1+i)
- $(1+i)F = A(1+i)^n + A(1+i)^{n-1} + \ldots + A(1+i)^3 + A(1+i)^2 + A(1+i)$
- Subtract 1st equation from 2nd
- $Fi = A[(1+i)^n - 1]$
- $F = A[((1+i)^n - 1)/i]$
- $F = A(F/A, i\%, n)$

8. Derivation of the A/F Equation

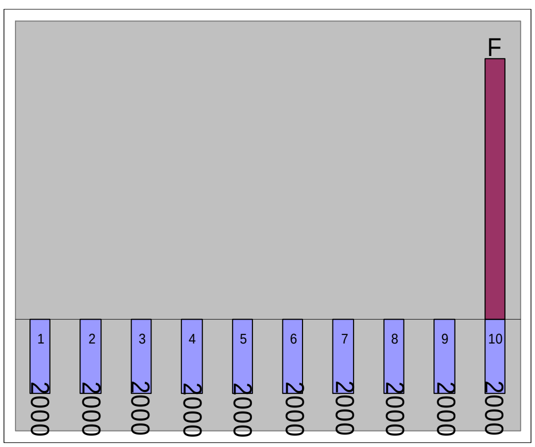
- Since $F = A[((1+i)^n - 1)/i]$
- Then $A = F[i/((1+i)^n - 1)]$

9. Derivation of the A/P and P/A Equations

- Given $F = P(1+i)^n$
- Given $A = F[i/((1+i)^n - 1)]$
- Substitute the 1st equation into the 2nd
- $A = P[[i(1+i)^n]/[(1+i)^n - 1]]$
- $P = A[[(1+i)^n - 1]/[i(1+i)^n]]$

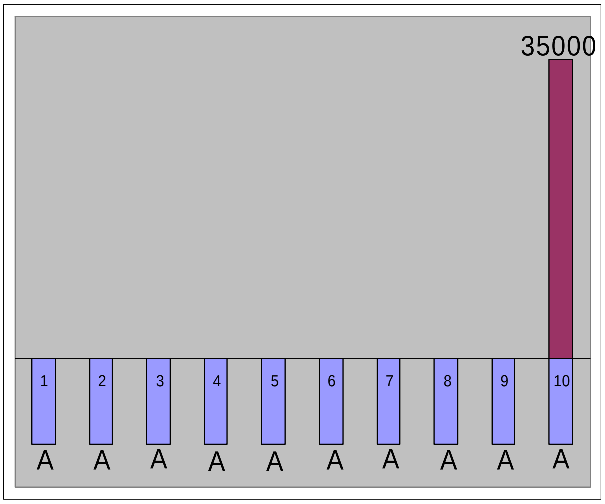
10. An Example of the F/A Factor

- Suppose your parents invested $2,000 per year for the 10 years prior to your high school graduation *(assume the last deposit occurred on the day of your graduation)*. At graduation, they gave you the money to help pay for your tuition. If they earned 7% per year on the investments, how much money did you have on the day of your graduation?



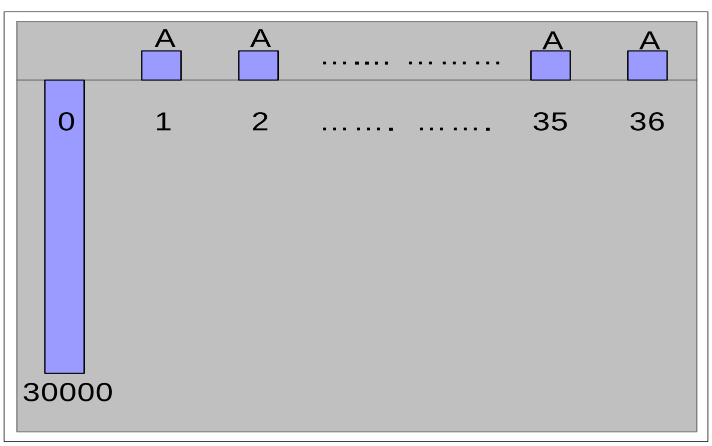
11. An Example of the A/F Factor

- In the previous example, your parents invested $2,000 per year. How much would they need to invest per year if they wanted $35,000 to be available to you at graduation? Again, interest is 7%.



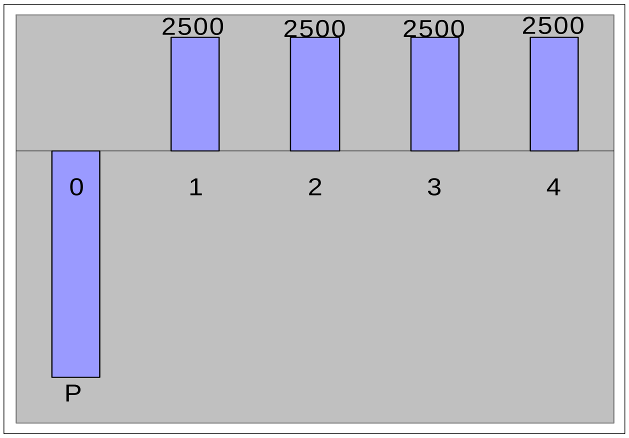
12. An Example of the A/P Factor

- A car that you desire costs $30,000. If interest, is 6% per year, compounded monthly, what will your monthly payments be if you decide to purchase the car? Payments are made every month for 3 years.



13. An Example of the P/A Factor

- How much must you invest today in order to be able to withdraw $2,500 per year for the next 4 years? Interest is 5%, compounded annually.



14. Arithmetic Gradient Factors that are Used to Move Money

- Find P, Given G (i.e., P/G)
- Find A, Given G (i.e., A/G)

15. What is an Arithmetic Gradient?

- A series of payments that ____
- A series of payments that is ___________________________________
- An arithmetic gradient is denoted with the capital letter *G*.

16. Find P, Given G (P/G, i%, n)

- This factor finds an equivalent single payment (i.e., P) from an arithmetic gradient (i.e., G).
- *P* is the point in time located *exactly* two periods prior to the first positive (or negative) dollar amount in the gradient. *Note: The first period of any gradient contains a zero dollar amount.*
- *0, G, 2G, …,* and *(n-1)G* are the series of points in time over which the money is currently located.
- *i%* is the effective interest rate for the period of time between any two consecutive uniformly increasing (or decreasing) payments.
- *n* is the number of uniformly increasing payments that are moved backwards. *Note: When determining the value of n, you must count the zero dollar ($0) amount that occurs at the first period.*

17. Find A, Given G (A/G, i%, n)

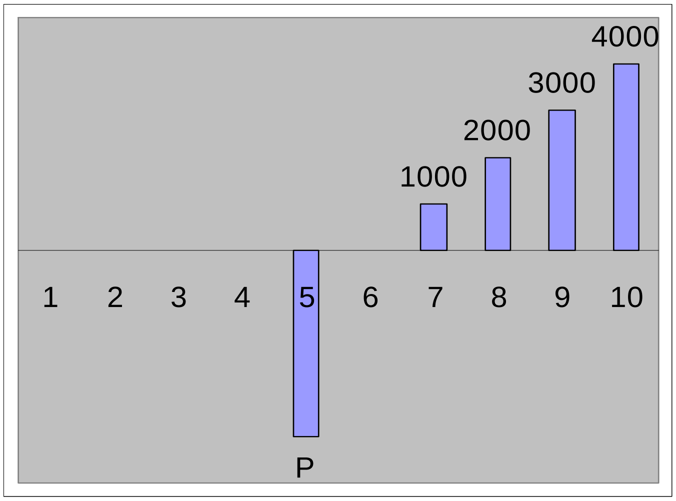
- This factor finds an equivalent uniform series of payments (i.e., A) from an arithmetic gradient (i.e., G).
- *A* is a uniform series of points in time that occurs *exactly* where the arithmetic gradient occurs. That is, it begins *exactly* where the first period in the gradient begins (i.e., the zero dollar amount) and ends *exactly* where the last growth in the gradient occurs.
- *0, G, 2G, …,* and *(n-1)G* are the series of points in time over which the money is currently located.
- *i%* is the effective interest rate for the period of time between any two consecutive uniformly increasing (or decreasing) payments.
- *n* is the number of equivalent uniform payments that are found from the arithmetic gradient. Because the uniform series occurs exactly where the arithmetic gradient is located, n is also equal to the number of periods in the arithmetic gradient. *Note: When determining the value of n based on the gradient, you must count the zero dollar ($0) amount that occurs at the first period.*

18. Derivation of P/G and A/G Equations

- We will not derive these equations. Please refer to your text for the equations.

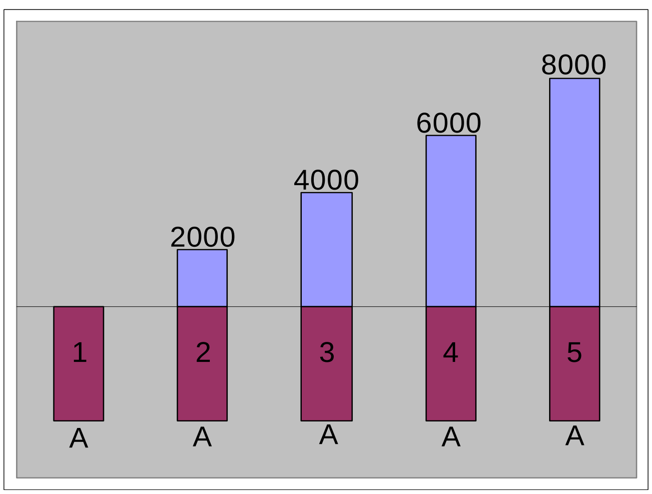
19. An Example of the P/G Factor

- In 5 years (i.e., at the end of year 5), you are planning on investing a lump sum amount of money. Your goal is to withdraw $1,000; $2,000; $3,000; and $4,000 at the end of years 7, 8, 9, and 10. Given an interest rate of 5%, what amount of money should be invested?



20. An Example of the A/G Factor

- If interest is 10% per numbered period on the graph, solve for A.



21. Geometric Gradient Factors that Are Used to Move Money

- Find P, Given A_1, when $i = g$ (i.e., P/A_1)
- Find P, Given A_1, when $i \neq g$ (i.e., P/A_1)

22. What is a Geometric Gradient?

- A geometric gradient occurs when the period-by-period change increases (or decreases) by a constant rate.

23. Find P/A_1 when $i = g$ $(P/A_1, g\%, i\%, n)$

- This factor finds an equivalent single payment (i.e., P) from a series of payments (i.e., A_1, A_2, …, A_n) that increase by a uniform interest rate (i.e., g).
- *P* is a point in time *one period* prior to where the first payment (i.e., A_1) occurs.
- $A_1, A_2, \ldots, A_n$ are a uniform series of points in time where the amounts of money are currently located.
- *g%* is the rate by which the payments increase from period to period.
- *i%* is the effective interest rate for the period of time between any two consecutive payments.
- *n* is the number of payments that are moved backwards.

24. Find P/A_1 when $i \neq g$ $(P/A_1, g\%, i\%, n)$

- This factor finds an equivalent single payment (i.e., P) from a series of payments (i.e., A_1, A_2, …, A_n) that increase by a uniform interest rate (i.e., g).
- *P* is a point in time *one period* prior to where the first payment (i.e., A_1) occurs.
- $A_1, A_2, \ldots, A_n$ are a uniform series of points in time where the amounts of money are currently located.
- *g%* is the rate by which the payments increase from period to period.
- *i%* is the effective interest rate for the period of time between any two consecutive payments.
- *n* is the number of payments that are moved backwards.

25. An Example of a Geometric Gradient

- Sally is going to invest $300 at the end of month #1 and will continue to invest money at the end of each month for 5 years (i.e., 60 months). Each monthly investment will be 1% more than the previous monthly investment. If interest is 1% per month, compounded monthly, what would be the equivalent amount of these cash flows today (i.e., the end of month 0)? What would be the equivalent value of the payments at the end of year 5 (i.e., month 60)?

26. Another Example of a Geometric Gradient

- How do the previous answers change if the interest is 0.5% per month instead of 1% per month?

27. Continuous Compounding

- If we increase *m*, the number of compounding sub-periods, without limit, *m* becomes very large and approaches infinity, and *r/m* becomes very small and approaches zero.

- Through a proof using calculus (see text), it can be shown that:

- $F = Pe^{(r)(n)}$ * r = nominal rate * n = number of periods moved at the nominal rate

- $P = Fe^{-(r)(n)}$

- $i_{eff} = e^{r} - 1$ where r = the nominal rate for the period of time for which you are looking for the effective rate

- You may omit any other continuous compounding equations.

28. Examples of Continuous Compounding

- Given that interest is 8% per year, compounded continuously, $2000 today is worth what amount in 5 years?

- Given that interest is 3% per half-year, compounded continuously, a $10000 payment that occurs five years from today is worth what amount today?

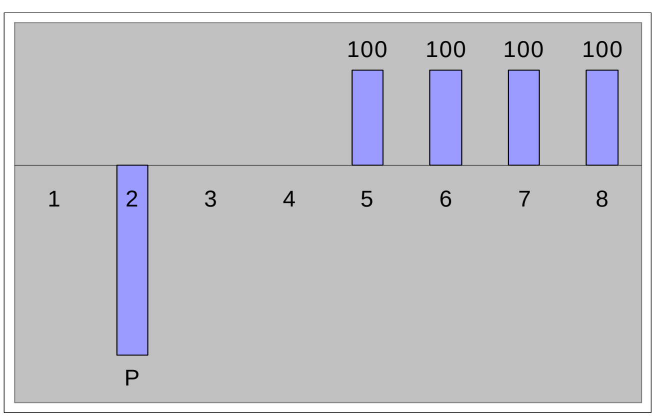
29. Before We Go Further, Let's Look at A Couple Key Terms

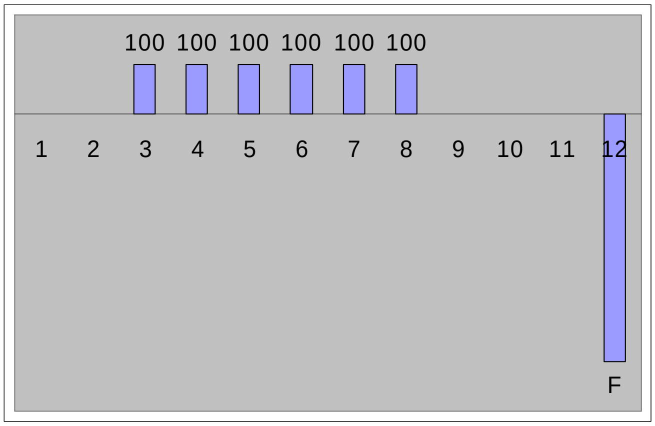
- The Minimum Attractive Rate of Return (MARR) is the ______________________________, or rate of return. You may be given either *i* or *MARR*. Treat them identically when making equivalency calculations.

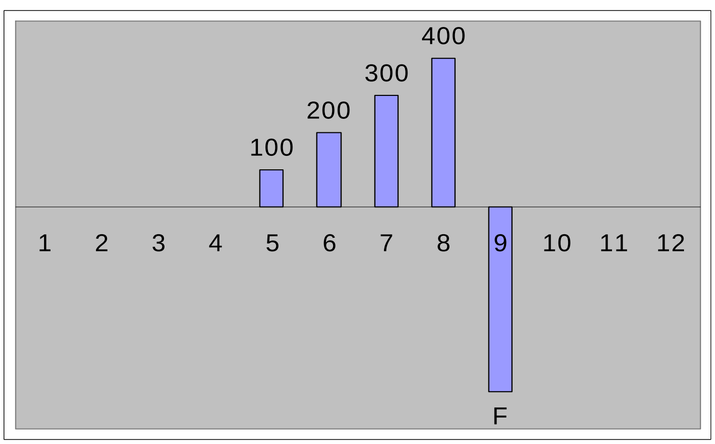
- A Sunk Cost is a __________________ that cannot be changed. It is, therefore, irrelevant for our calculations in this course. The one exception is that a sunk cost may be used to calculate an asset's depreciation schedule needed for after-tax analyses.

30. More Examples

- Use an MARR of 10% per year, compounded annually for each of the following graphs.
- For each of the following graphs, the numbered periods represent years.




100 100 100 100 100 100
1 2 3 4 5 6 7 8 9 10 11 12
F


400
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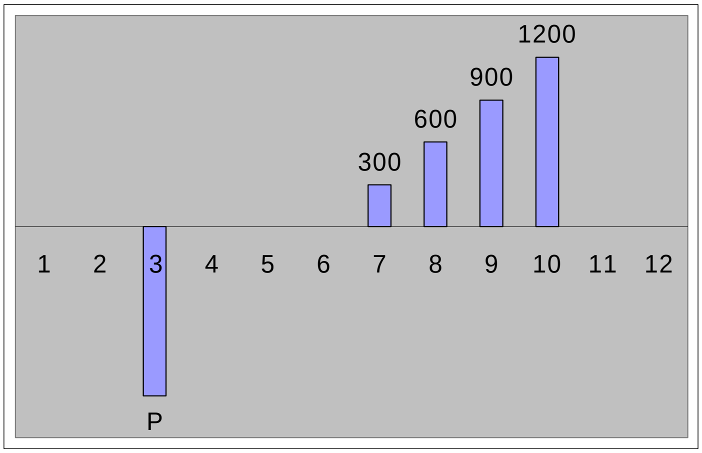
1 2 3 4 5 6 7 8 9 10 11 12
F


1200
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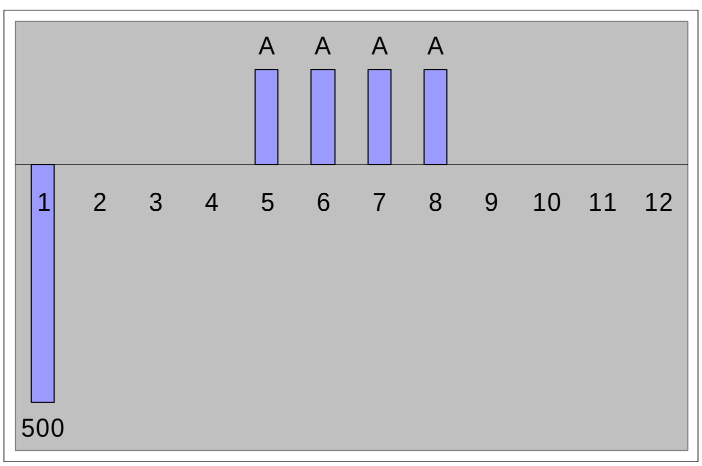
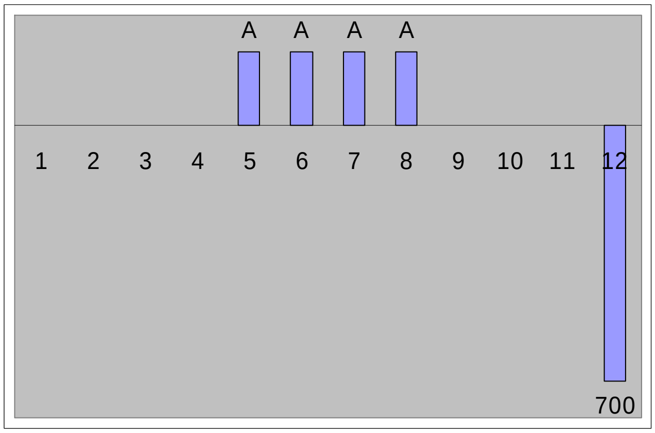
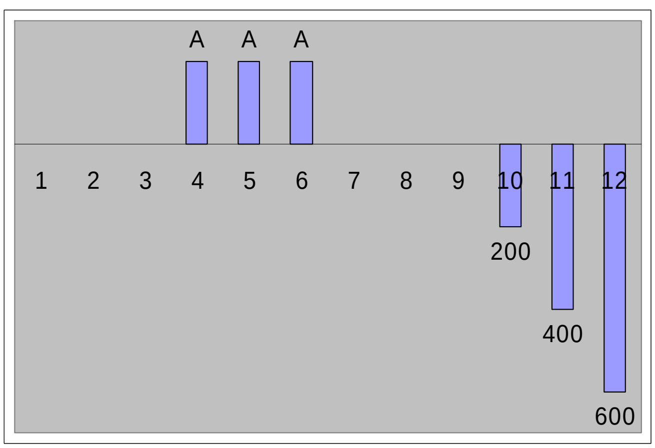
1 2 3 4 5 6 7 8 9 10 11 12
P


A A A A
1 2 3 4 5 6 7 8 9 10 11 12
500


A A A A
1 2 3 4 5 6 7 8 9 10 11 12
700


A A A
1 2 3 4 5 6 7 8 9 10 11 12
200
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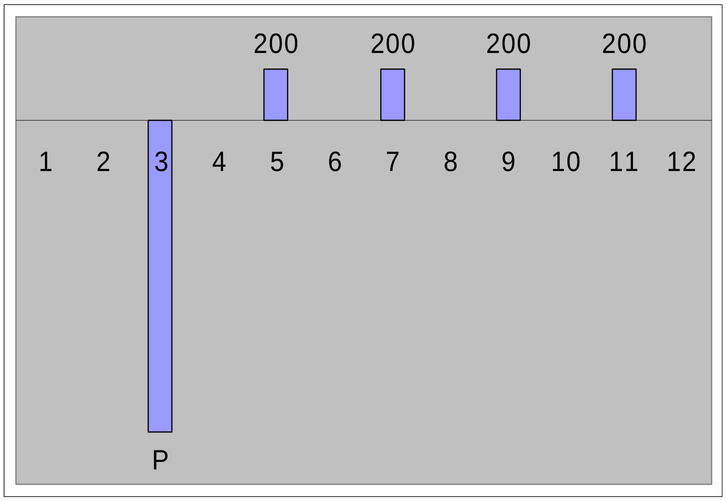
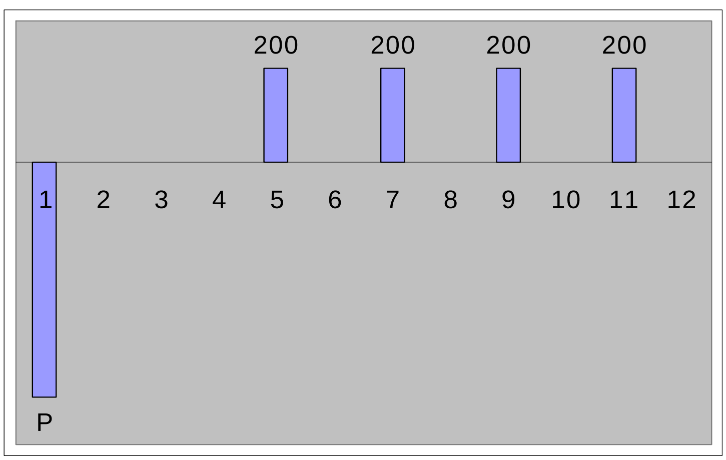


- Since the uniform series is spaced every two year, we need the effective two-year interest rate.



- Use the same rationale as the previous problem.